Chapter 1
Introduction

Learning Objectives

1. Develop a general understanding of the management science/operations research approach to decision making.

2. Realize that quantitative applications begin with a problem situation.

3. Obtain a brief introduction to quantitative techniques and their frequency of use in practice.

4. Understand that managerial problem situations have both quantitative and qualitative considerations that are important in the decision making process.

5. Learn about models in terms of what they are and why they are useful (the emphasis is on mathematical models).

6. Identify the step-by-step procedure that is used in most quantitative approaches to decision making.

7. Learn about basic models of cost, revenue, and profit and be able to compute the breakeven point.

8. Obtain an introduction to the use of computer software packages such as *Microsoft Excel* in applying quantitative methods to decision making.

9. Understand the following terms:

model	infeasible solution
objective function	management science
constraint	operations research
deterministic model	fixed cost
stochastic model	variable cost
feasible solution	breakeven point

Solutions:

1. Management science and operations research, terms used almost interchangeably, are broad disciplines that employ scientific methodology in managerial decision making or problem solving. Drawing upon a variety of disciplines (behavioral, mathematical, etc.), management science and operations research combine quantitative and qualitative considerations in order to establish policies and decisions that are in the best interest of the organization.

2. Define the problem

   Identify the alternatives

   Determine the criteria

   Evaluate the alternatives

   Choose an alternative

   For further discussion see section 1.3

3. See section 1.2.

4. A quantitative approach should be considered because the problem is large, complex, important, new and repetitive.

5. Models usually have time, cost, and risk advantages over experimenting with actual situations.

6. Model (a) may be quicker to formulate, easier to solve, and/or more easily understood.

7. Let d = distance
   m = miles per gallon
   c = cost per gallon,

   $$\therefore \text{Total Cost} = \left(\frac{2d}{m}\right)c$$

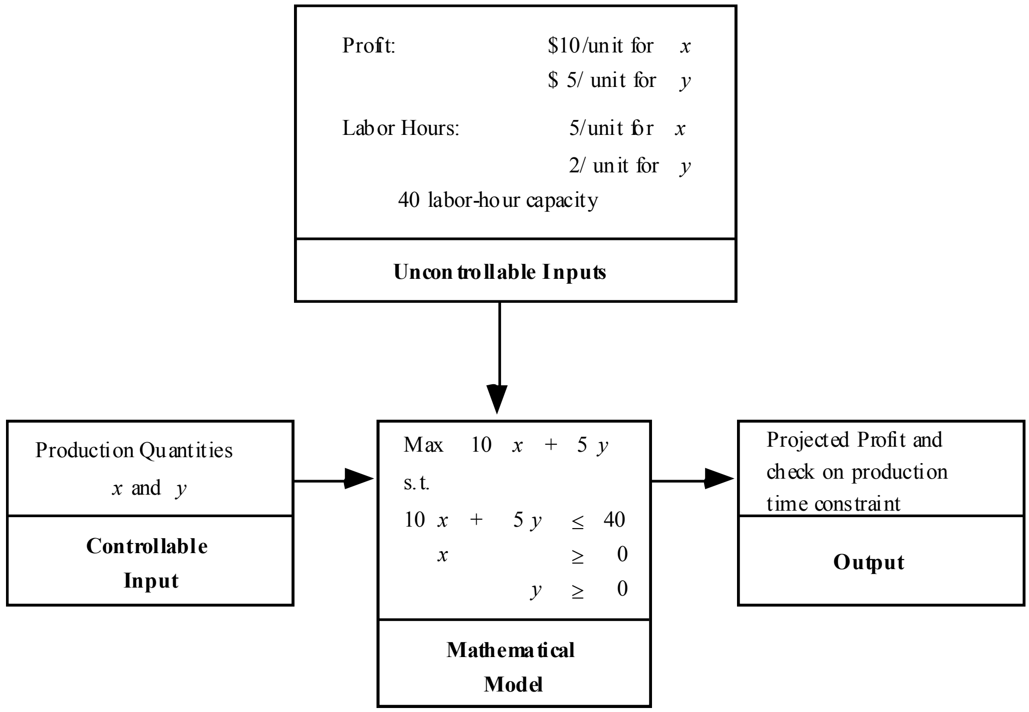
   We must be willing to treat m and c as known and not subject to variation.

8. a. Maximize $10x + 5y$
   s.t.

   $$5x + 2y \le 40$$
   $$x \ge 0, y \ge 0$$

   b. Controllable inputs: x and y
   Uncontrollable inputs: profit (10,5), labor hours (5,2) and labor-hour availability (40)

   c.



d. $x = 0$, $y = 20$ Profit = \$100
(Solution by trial-and-error)

e. Deterministic - all uncontrollable inputs are fixed and known.

9. If $a = 3$, $x = 13\ 1/3$ and profit = 133
If $a = 4$, $x = 10$ and profit = 100
If $a = 5$, $x = 8$ and profit = 80
If $a = 6$, $x = 6\ 2/3$ and profit = 67

Since a is unknown, the actual values of x and profit are not known with certainty.

10. a. Total Units Received = $x + y$

b. Total Cost = $0.20x + 0.25y$

c. $x + y = 5000$

d. $x \leq 4000$ Kansas City Constraint
$y \leq 3000$ Minneapolis Constraint

e. Min $0.20x + 0.25y$
s.t.

$$\begin{aligned} x + y &= 5000 \\ x &\leq 4000 \\ y &\leq 3000 \\ \\ x, y &\geq 0 \end{aligned}$$

11. a. at \$20 $d = 800 - 10(20) = 600$
at \$70 $d = 800 - 10(70) = 100$

b. at \$26 $d = 800 - 10(26) = 540$
at \$27 $d = 800 - 10(27) = 530$
If the firm increases the per unit price from \$26 to \$27, the number of units the firm can sell falls by 10.

at \$42 $d = 800 - 10(42) = 380$
at \$43 $d = 800 - 10(43) = 370$
If the firm increases the per unit price from \$426 to \$43, the number of units the firm can sell falls by 10.

This suggests that the relationship between the per-unit price and annual demand for the product in units is linear between \$20 and \$70 and that annual demand for the product decreases by 10 units when the price is increased by \$1.

c. $TR = dp = (800 - 10p)p = 800p - 10p^2$

d. at \$30 $TR = 800(30) - 10(30)^2 = 15{,}000$
at \$40 $TR = 800(40) - 10(40)^2 = 16{,}000$
at \$50 $TR = 800(50) - 10(50)^2 = 15{,}000$
Total Revenue is maximized at the \$40 price.

e. $d = 800 - 10(40) = 400$ units
$TR = \$16{,}000$

12. a. $TC = 2000 + 60x$

b. $P = 80x - (2000 + 60x) = 20x - 2000$

c. Breakeven point is the value of x when $P = 0$
Thus $20x - 2000 = 0$
$20x = 2000$
$x = 100$

13. a. Total cost = $9600 + (2*60)x = 9600 + 120x$

b. Total profit = total revenue - total cost
= $600x - (9600 + 120x)$
= $480x - 9600$

c. Total profit = $480(30) - 9600 = 4800$

d. $480x - 9600 = 0$

$x = 9600/480 = 20$

The breakeven point is 20 students.

14. a. Profit = Revenue - Cost
= $46x - (160{,}000 + 6x)$
= $40x - 160{,}000$

$40x - 160{,}000 = 0$
$40x = 160{,}000$
$x = 4000$

Breakeven point = 4000

b. Profit = 40(3800) - 16,000 = -8000

Thus, a loss of $8000 is anticipated.

c. Profit = px - (160,000 + 6x)
= 3800p - (160,000 + 6(3800)) = 0
3800p = 182,800
p = 48.105 or $48.11

d. Profit = $50.95 (3800) - (160,000 + 6 (3800))
= $10,810

Probably go ahead with the project although the $10,810 is only a 5.98% return on the total cost of $182,800.

15. a. Profit = 300,000x - (4,500,000 + 150,000x) = 0
150,000x = 4,500,000
x = 30

b. Build the luxury boxes.

Profit = 300,000 (50) - (4,500,000 + 150,000 (50))
= $3,000,000

16. a. Max 6x + 4y

b.
$$50x + 30y \leq 800,000$$
$$50x \leq 500,000$$
$$30y \leq 450,000$$
$$x, y \geq 0$$

17. a. $s_j = s_{j-1} + x_j - d_j$

or $s_j - s_{j-1} - x_j + d_j = 0$

b. $x_j \leq c_j$

c. $s_j \geq I_j$

18. a. maximize $(3.10 - 0.30)x_1 + (3.10 - 0.40)\,x_2 + (3.10 - 0.48)x_3 + (3.20 - 0.30)y_1 + (3.20 - 0.40)y_2$ $+(3.20\text{-}0.48)y_3$
= maximize $2.80x_1 + 2{,}70\,x_2 + 2.62x_3 + 2.90y_1 + 2.80y_2 + 2.72y_3$

b. (1) $x_1 + y_1 \leq 12,000$
(2) $x_2 + y_2 \leq 20,000$
(3) $x_3 + y_3 \leq 24,000$

c. $x_1 \geq .35(x_1 + x_2 + x_3)$ or $.65x_1 - .35x_2 - .35x_3 \geq 0$ (Blend A must be composed of at least 35% of oil from the Texas well)
$x_2 \leq .50(x_1 + x_2 + x_3)$ or $-.50x_1 + .50x_2 - .50x_3 \leq 0$ (Blend A must be composed of no more than 50% of oil from the Oklahoma well)
$x_3 \geq .15(x_1 + x_2 + x_3)$ or $-.15x_1 - .15x_2 + .85x_3 \geq 0$ (Blend A must be composed of at least 15% of oil from the California well)
$y_1 \geq .20(y_1 + y_2 + y_3)$ or $.80y_1 - .20y_2 - .20y_3 \geq 0$ (Blend B must be composed of at least 20% of oil from the Texas well)

$y_2 \geq .30(y_1 + y_2 + y_3)$ or $-.30y_1 + .70y_2 - .30y_3 \geq 0$	(Blend B must be composed of at least 30% of oil from the Oklahoma well)
$y_3 \leq .40(y_1 + y_2 + y_3)$ or $-.40y_1 - .40y_2 + .60y_3 \leq 0$	(Blend B must be composed of no more than 40% of oil from the California well)
$x_1 + x_2 + x_3 \geq 20{,}000$	(Long-term contracts require at least 20,000 gallons of Blend A to be produced)
$y_1 + y_2 + y_3 \geq 20{,}000$	(Long-term contracts require at least 20,000 gallons of Blend B to be produced)

19. a. Profit per contemporary cabinet is \$90 – [2.0(\$15) + 1.5(\$12) + 1.3(\$18)] = \$18.60
Profit per farmhouse cabinet is \$85 – [2.5(\$15) + 1.0(\$12) + 1.2(\$18)] = \$13.90
So the objective function is maximize $18.6x + 13.9y$

b. $2.0x_1 + 2.5y_1 \leq 3000$ hours available in carpentry
$1.5x_2 + 1.0y_2 \leq 1500$ hours available in carpentry
$1.3x_3 + 1.2y_3 \leq 1500$ hours available in carpentry

c. $x \geq 500$
$y \geq 650$

20. a. maximize $7000x + 4000y$

b. $500x + 250y \leq 100{,}000$

c. $x \leq 20$

d. $y \geq 50$

e. $\frac{x}{x+y} \geq \frac{1}{3}$, or $2x - y \geq 0$

f. If the constraints for the maximum number of television ads to be used from part(c) and the minimum number of internet ads to be used from part (d) are both satisfied, then television ads can be at most $\frac{20}{20+50} = 0.285 \leq 0.333$. That is, the constraint for the stipulated ratio of television ads to Internet ads cannot be satisfied. Therefore, the problem as stated is infeasible.

Chapter 1
Introduction

Case Problem: Scheduling a Golf League

Note to Instructor: This case problem illustrates the value of the rational management science approach. The problem is easy to understand and, at first glance, appears simple. But, most students will have trouble finding a solution. The solution procedure suggested involves decomposing a larger problem into a series of smaller problems that are easier to solve. The case provides students with a good first look at the kinds of problems where management science is applied in practice. The problem is a real one that one of the authors was asked by the Head Professional at Royal Oak Country Club for help with.

Solution: Scheduling problems such as this occur frequently, and are often difficult to solve. The typical approach is to use trial and error. An alternative approach involves breaking the larger problem into a series of smaller problems. We show how this can be done here using what we call the Red, White, and Blue algorithm.

Suppose we break the 18 couples up into 3 divisions, referred to as the Red, White, and Blue divisions. The six couples in the Red division can then be identified as R1, R2, R3, R4, R5, R6; the six couples in the White division can be identified as W1, W2,…, W6; and the six couples in the Blue division can be identified as B1, B2,…, B6. We begin by developing a schedule for the first 5 weeks of the season so that each couple plays every other couple in its own division. This can be done fairly easily by trial and error. Shown below is the first 5-week schedule for the Red division.

Week 1	**Week 2**	**Week 3**	**Week 4**	**Week 5**
R1 vs. R2	R1 vs. R3	R1 vs. R4	R1 vs. R5	R1 vs. R6
R3 vs. R4	R2 vs. R5	R2 vs. R6	R2 vs. R4	R2 vs. R3
R5 vs. R6	R4 vs. R6	R3 vs. R5	R3 vs. R6	R4 vs. R5

Similar 5-week schedules can be developed for the White and Blue divisions by replacing the R in the above table with a W or a B.

To develop the schedule for the next 3 weeks, we create 3 new six-couple divisions by pairing 3 of the teams in each division with 3 of the teams in another division; for example, (R1, R2, R3, W1, W2, W3), (B1, B2, B3, R4, R5, R6), and (W4, W5, W6, B4, B5, B6). Within each of these new divisions, matches can be scheduled for 3 weeks without any couples playing a couple they have played before. For instance, a 3-week schedule for the first of these divisions is shown below:

Week 6	**Week 7**	**Week 8**
R1 vs. W1	R1 vs. W2	R1 vs. W3
R2 vs. W2	R2 vs. W3	R2 vs. W1
R3 vs. W3	R3 vs. W1	R3 vs. W2

A similar 3-week schedule can be easily set up for the other two new divisions. This will provide us with a schedule for the first 8 weeks of the season.

For the final 9 weeks, we continue to create new divisions by pairing 3 teams from the original Red, White and Blue divisions with 3 teams from the other divisions that they have not yet been paired with. Then a 3-week schedule is developed as above. Shown below is a set of divisions for the next 9 weeks.

Weeks 9-11

(R1, R2, R3, W4, W5, W6)	(W1, W2, W3, B1, B2, B3)	(R4, R5, R6, B4, B5, B6)

Weeks 12-14

(R1, R2, R3, B1, B2, B3)	(W1, W2, W3, B4, B5, B6)	(W4, W5, W6, R4, R5, R6)

Weeks 15-17

(R1, R2, R3, B4, B5, B6)	(W1, W2, W3, R4, R5, R6)	(W4, W5, W6, B1, B2, B3)

This Red, White and Blue scheduling procedure provides a schedule with every couple playing every other couple over the 17-week season. If one of the couples should cancel, the schedule can be modified easily. Designate the couple that cancels, say R4, as the Bye couple. Then whichever couple is scheduled to play couple R4 will receive a Bye in that week. With only 17 couples a Bye must be scheduled for one team each week.

This same scheduling procedure can obviously be used for scheduling sports teams and or any other kinds of matches involving 17 or 18 teams. Modifications of the Red, White and Blue algorithm can be employed for 15 or 16 team leagues and other numbers of teams.