INSTRUCTOR'S SOLUTIONS MANUAL

DUANE KOUBA
University of California, Davis

JENNIFER A. BLUE
SUNY Empire State College

UNIVERSITY CALCULUS EARLY TRANSCENDENTALS

FOURTH EDITION

Joel Hass
University of California, Davis

Christopher Heil
Georgia Institute of Technology

Przemyslaw Bogacki
Old Dominion University

Maurice D. Weir
Naval Postgraduate School

George B. Thomas, Jr.
Massachusetts Institute of Technology

Pearson

This work is protected by United States copyright laws and is provided solely for the use of instructors in teaching their courses and assessing student learning. Dissemination or sale of any part of this work (including on the World Wide Web) will destroy the integrity of the work and is not permitted. The work and materials from it should never be made available to students except by instructors using the accompanying text in their classes. All recipients of this work are expected to abide by these restrictions and to honor the intended pedagogical purposes and the needs of other instructors who rely on these materials.

The author and publisher of this book have used their best efforts in preparing this book. These efforts include the development, research, and testing of the theories and programs to determine their effectiveness. The author and publisher make no warranty of any kind, expressed or implied, with regard to these programs or the documentation contained in this book. The author and publisher shall not be liable in any event for incidental or consequential damages in connection with, or arising out of, the furnishing, performance, or use of these programs.

Reproduced by Pearson from electronic files supplied by the author.

Copyright © 2020, 2016, 2012 by Pearson Education, Inc. 221 River Street, Hoboken, NJ 07030. All rights reserved.

All rights reserved. No part of this publication may be reproduced, stored in a retrieval system, or transmitted, in any form or by any means, electronic, mechanical, photocopying, recording, or otherwise, without the prior written permission of the publisher. Printed in the United States of America.

ISBN-13: 978-0-13-516571-3
ISBN-10: 0-13-516571-7

TABLE OF CONTENTS

1 Functions 1

1.1 Functions and Their Graphs 1
1.2 Combining Functions; Shifting and Scaling Graphs 9
1.3 Trigonometric Functions 19
1.4 Graphing with Software 27
1.5 Exponential Functions 32
1.6 Inverse Functions and Logarithms 35

2 Limits and Continuity 47

2.1 Rates of Change and Tangent Lines to Curves 47
2.2 Limit of a Function and Limit Laws 51
2.3 The Precise Definition of a Limit 61
2.4 One-Sided Limits 70
2.5 Continuity 75
2.6 Limits Involving Infinity; Asymptotes of Graphs 81
Practice Exercises 92
Additional and Advanced Exercises 98

3 Derivatives 107

3.1 Tangent Lines and the Derivative at a Point 107
3.2 The Derivative as a Function 113
3.3 Differentiation Rules 124
3.4 The Derivative as a Rate of Change 130
3.5 Derivatives of Trigonometric Functions 136
3.6 The Chain Rule 143
3.7 Implicit Differentiation 155
3.8 Derivatives of Inverse Functions and Logarithms 163
3.9 Inverse Trigonometric Functions 173
3.10 Related Rates 180
3.11 Linearization and Differentials 185
Practice Exercises 193
Additional and Advanced Exercises 207

Copyright © 2020 Pearson Education, Inc.

4 Applications of Derivatives 213

4.1 Extreme Values of Functions on Closed Intervals 213
4.2 The Mean Value Theorem 222
4.3 Monotonic Functions and the First Derivative Test 229
4.4 Concavity and Curve Sketching 243
4.5 Indeterminate Forms and L'Hôpital's Rule 267
4.6 Applied Optimization 276
4.7 Newton's Method 291
4.8 Antiderivatives 296
Practice Exercises 306
Additional and Advanced Exercises 327

5 Integrals 335

5.1 Area and Estimating with Finite Sums 335
5.2 Sigma Notation and Limits of Finite Sums 340
5.3 The Definite Integral 346
5.4 The Fundamental Theorem of Calculus 361
5.5 Indefinite Integrals and the Substitution Method 372
5.6 Definite Integral Substitutions and the Area Between Curves 380
Practice Exercises 399
Additional and Advanced Exercises 415

6 Applications of Definite Integrals 421

6.1 Volumes Using Cross-Sections 421
6.2 Volumes Using Cylindrical Shells 433
6.3 Arc Length 445
6.4 Areas of Surfaces of Revolution 454
6.5 Work and Fluid Forces 459
6.6 Moments and Centers of Mass 466
Practice Exercises 479
Additional and Advanced Exercises 489

7 Integrals and Transcendental Functions 495

7.1 The Logarithm Defined as an Integral 495
7.2 Exponential Change and Separable Differential Equations 502
7.3 Hyperbolic Functions 507
Practice Exercises 516
Additional and Advanced Exercises 520

Copyright © 2020 Pearson Education, Inc.

8 Techniques of Integration 523

8.1 Integration by Parts 523
8.2 Trigonometric Integrals 537
8.3 Trigonometric Substitutions 546
8.4 Integration of Rational Functions by Partial Fractions 556
8.5 Integral Tables and Computer Algebra Systems 566
8.6 Numerical Integration 578
8.7 Improper Integrals 588
Practice Exercises 601
Additional and Advanced Exercises 615

9 Infinite Sequences and Series 623

9.1 Sequences 623
9.2 Infinite Series 635
9.3 The Integral Test 644
9.4 Comparison Tests 652
9.5 Absolute Convergence; The Ratio and Root Tests 663
9.6 Alternating Series and Conditional Convergence 669
9.7 Power Series 679
9.8 Taylor and Maclaurin Series 691
9.9 Convergence of Taylor Series 697
9.10 The Binomial Series and Applications of Taylor Series 705
Practice Exercises 714
Additional and Advanced Exercises 725

10 Parametric Equations and Polar Coordinates 731

10.1 Parametrizations of Plane Curves 731
10.2 Calculus with Parametric Curves 737
10.3 Polar Coordinates 747
10.4 Graphing Polar Coordinate Equations 753
10.5 Areas and Lengths in Polar Coordinates 761
Practice Exercises 767
Additional and Advanced Exercises 774

Copyright © 2020 Pearson Education, Inc.

11 Vectors and the Geometry of Space 777

11.1 Three-Dimensional Coordinate Systems 777
11.2 Vectors 782
11.3 The Dot Product 788
11.4 The Cross Product 794
11.5 Lines and Planes in Space 801
11.6 Cylinders and Quadric Surfaces 809
Practice Exercises 815
Additional and Advanced Exercises 823

12 Vector-Valued Functions and Motion in Space 829

12.1 Curves in Space and Their Tangents 829
12.2 Integrals of Vector Functions; Projectile Motion 836
12.3 Arc Length in Space 842
12.4 Curvature and Normal Vectors of a Curve 846
12.5 Tangential and Normal Components of Acceleration 853
12.6 Velocity and Acceleration in Polar Coordinates 857
Practice Exercises 859
Additional and Advanced Exercises 866

13 Partial Derivatives 869

13.1 Functions of Several Variables 869
13.2 Limits and Continuity in Higher Dimensions 879
13.3 Partial Derivatives 887
13.4 The Chain Rule 896
13.5 Directional Derivatives and Gradient Vectors 906
13.6 Tangent Planes and Differentials 912
13.7 Extreme Values and Saddle Points 921
13.8 Lagrange Multipliers 937
Practice Exercises 949
Additional and Advanced Exercises 965

Copyright © 2020 Pearson Education, Inc.

14 Multiple Integrals 971

14.1 Double and Iterated Integrals over Rectangles 971
14.2 Double Integrals over General Regions 974
14.3 Area by Double Integration 988
14.4 Double Integrals in Polar Form 993
14.5 Triple Integrals in Rectangular Coordinates 999
14.6 Applications 1005
14.7 Triple Integrals in Cylindrical and Spherical Coordinates 1011
14.8 Substitutions in Multiple Integrals 1024
Practice Exercises 1031
Additional and Advanced Exercises 1039

15 Integrals and Vector Fields 1045

15.1 Line Integrals 1045
15.2 Vector Fields and Line Integrals: Work, Circulation, and Flux 1051
15.3 Path Independence, Conservative Fields, and Potential Functions 1063
15.4 Green's Theorem in the Plane 1069
15.5 Surfaces and Area 1077
15.6 Surface Integrals 1087
15.7 Stokes' Theorem 1098
15.8 The Divergence Theorem and a Unified Theory 1105
Practice Exercises 1112
Additional and Advanced Exercises 1122

16 First-Order Differential Equations 1127

16.1 Solutions, Slope Fields, and Euler's Method 1127
16.2 First-Order Linear Equations 1137
16.3 Applications 1141
16.4 Graphical Solutions of Autonomous Equations 1145
16.5 Systems of Equations and Phase Planes 1152
Practice Exercises 1158
Additional and Advanced Exercises 1166

Copyright © 2020 Pearson Education, Inc.

17 Second-Order Differential Equations 1169

17.1 Second-Order Linear Equations 1169
17.2 Nonhomogeneous Linear Equations 1174
17.3 Applications 1182
17.4 Euler Equations 1186
17.5 Power-Series Solutions 1189

B Appendix 1197

B.1 Relative Rates of Growth 1197
B.2 Probability 1202
B.3 Conics in Polar Coordinates 1210
B.4 Taylor's Formula for Two Variables 1220
B.5 Partial Derivatives with Constrained Variables 1223

Copyright © 2020 Pearson Education, Inc.

CHAPTER 1 FUNCTIONS

1.1 FUNCTIONS AND THEIR GRAPHS

1. domain = $(-\infty, \infty)$; range = $[1, \infty)$

2. domain = $[0, \infty)$; range = $(-\infty, 1]$

3. domain = $[-2, \infty)$; y in range and $y = \sqrt{5x+10} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range = $[0, \infty)$.

4. domain = $(-\infty, 0] \cup [3, \infty)$; y in range and $y = \sqrt{x^2 - 3x} \geq 0 \Rightarrow$ y can be any positive real number $\Rightarrow$ range = $[0, \infty)$.

5. domain = $(-\infty, 3) \cup (3, \infty)$; y in range and $y = \frac{4}{3-t}$, now if $t < 3 \Rightarrow 3 - t > 0 \Rightarrow \frac{4}{3-t} > 0$, or if $t > 3 \Rightarrow 3 - t < 0 \Rightarrow \frac{4}{3-t} < 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range = $(-\infty, 0) \cup (0, \infty)$.

6. domain = $(-\infty, -4) \cup (-4, 4) \cup (4, \infty)$; y in range and $y = \frac{2}{t^2 - 16}$, now if $t < -4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0$, or if $-4 < t < 4 \Rightarrow -16 \leq t^2 - 16 < 0 \Rightarrow -\frac{2}{16} \geq \frac{2}{t^2-16}$, or if $t > 4 \Rightarrow t^2 - 16 > 0 \Rightarrow \frac{2}{t^2-16} > 0 \Rightarrow$ y can be any nonzero real number $\Rightarrow$ range = $(-\infty, -\frac{1}{8}] \cup (0, \infty)$.

7. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Is the graph of a function of x since any vertical line intersects the graph at most once.

8. (a) Not the graph of a function of x since it fails the vertical line test.
   (b) Not the graph of a function of x since it fails the vertical line test.

9. base = x; $(\text{height})^2 + \left(\frac{x}{2}\right)^2 = x^2 \Rightarrow$ height = $\frac{\sqrt{3}}{2}x$; area is $a(x) = \frac{1}{2}$ (base)(height) $= \frac{1}{2}(x)\left(\frac{\sqrt{3}}{2}x\right) = \frac{\sqrt{3}}{4}x^2$; perimeter is $p(x) = x + x + x = 3x$.

10. s = side length $\Rightarrow s^2 + s^2 = d^2 \Rightarrow s = \frac{d}{\sqrt{2}}$; and area is $a = s^2 \Rightarrow a = \frac{1}{2}d^2$

11. Let D = diagonal length of a face of the cube and ℓ = the length of an edge. Then $\ell^2 + D^2 = d^2$ and $D^2 = 2\ell^2 \Rightarrow 3\ell^2 = d^2 \Rightarrow \ell = \frac{d}{\sqrt{3}}$. The surface area is $6\ell^2 = \frac{6d^2}{3} = 2d^2$ and the volume is $\ell^3 = \left(\frac{d^2}{3}\right)^{3/2} = \frac{d^3}{3\sqrt{3}}$.

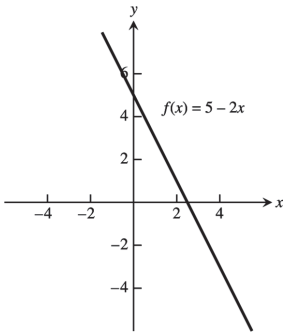
12. The coordinates of P are $\left(x, \sqrt{x}\right)$ so the slope of the line joining P to the origin is $m = \frac{\sqrt{x}}{x} = \frac{1}{\sqrt{x}} (x > 0)$. Thus, $\left(x, \sqrt{x}\right) = \left(\frac{1}{m^2}, \frac{1}{m}\right)$.

13. $2x + 4y = 5 \Rightarrow y = -\frac{1}{2}x + \frac{5}{4}; L = \sqrt{(x-0)^2 + (y-0)^2} = \sqrt{x^2 + (-\frac{1}{2}x + \frac{5}{4})^2} = \sqrt{x^2 + \frac{1}{4}x^2 - \frac{5}{4}x + \frac{25}{16}}$
$= \sqrt{\frac{5}{4}x^2 - \frac{5}{4}x + \frac{25}{16}} = \sqrt{\frac{20x^2 - 20x + 25}{16}} = \frac{\sqrt{20x^2 - 20x + 25}}{4}$

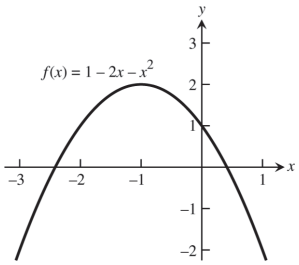
14. $y = \sqrt{x-3} \Rightarrow y^2 + 3 = x; L = \sqrt{(x-4)^2 + (y-0)^2} = \sqrt{(y^2 + 3 - 4)^2 + y^2} = \sqrt{(y^2 - 1)^2 + y^2}$
$= \sqrt{y^4 - 2y^2 + 1 + y^2} = \sqrt{y^4 - y^2 + 1}$

Copyright © 2020 Pearson Education, Inc.

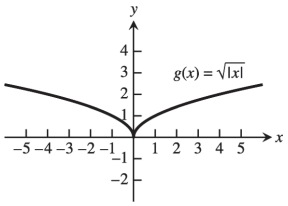
15. The domain is $(-\infty, \infty)$.



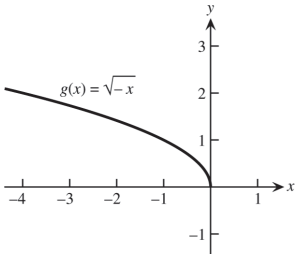
16. The domain is $(-\infty, \infty)$.



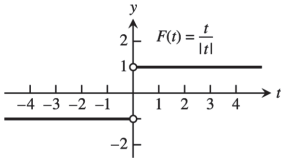
17. The domain is $(-\infty, \infty)$.



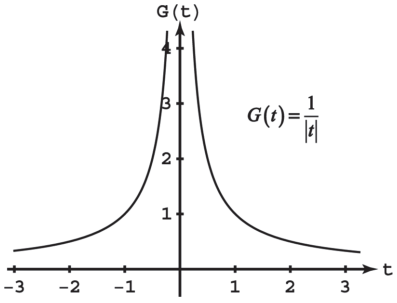
18. The domain is $(-\infty, 0]$.



19. The domain is $(-\infty, 0) \cup (0, \infty)$.



20. The domain is $(-\infty, 0) \cup (0, \infty)$.

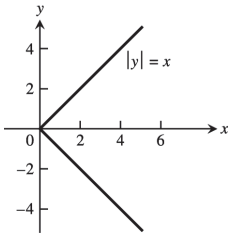


21. The domain is $(-\infty, -5) \cup (-5, -3] \cup [3, 5) \cup (5, \infty)$

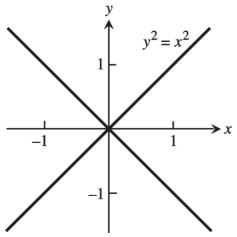
22. The range is $[5, \infty)$.

23. Neither graph passes the vertical line test

(a)



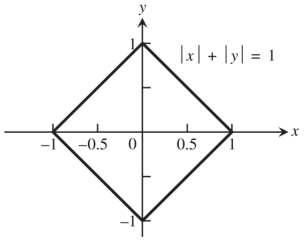
(b)



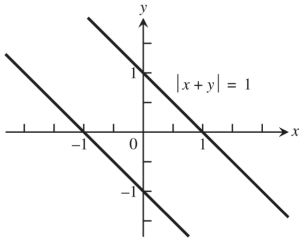
Copyright © 2020 Pearson Education, Inc.

24. Neither graph passes the vertical line test

(a)



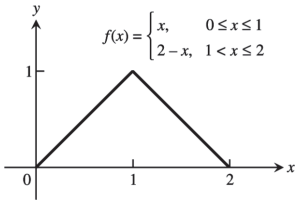
(b)



$$|x+y|=1 \Leftrightarrow \left\{\begin{array}{c} x+y=1 \\ \text{or} \\ x+y=-1 \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} y=1-x \\ \text{or} \\ y=-1-x \end{array}\right\}$$

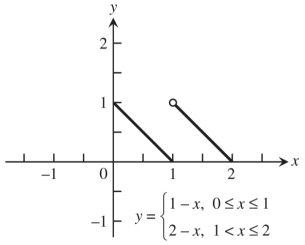
25.

x	0	1	2
y	0	1	0

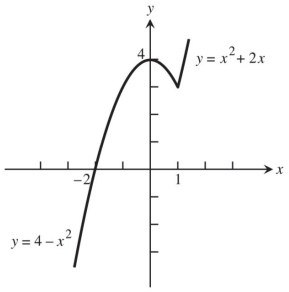


26.

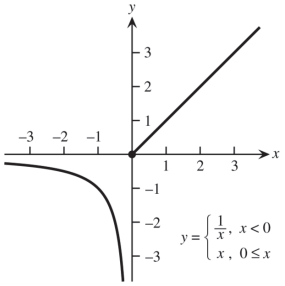
x	0	1	2
y	1	0	0



27. $F(x)=\begin{cases} 4-x^2, & x \le 1 \\ x^2+2x, & x>1 \end{cases}$



28. $G(x)=\begin{cases} \frac{1}{x}, & x<0 \\ x, & 0 \le x \end{cases}$



29. (a) Line through (0, 0) and (1, 1): $y = x$; Line through (1, 1) and (2, 0): $y = -x + 2$

$$f(x)=\begin{cases} x, & 0 \le x \le 1 \\ -x+2, & 1<x \le 2 \end{cases}$$

(b) $f(x)=\begin{cases} 2, & 0 \le x<1 \\ 0, & 1 \le x<2 \\ 2, & 2 \le x<3 \\ 0, & 3 \le x \le 4 \end{cases}$

30. (a) Line through (0, 2) and (2, 0): $y = -x + 2$

Line through (2, 1) and (5, 0): $m=\frac{0-1}{5-2}=\frac{-1}{3}=-\frac{1}{3}$, so $y=-\frac{1}{3}(x-2)+1=-\frac{1}{3}x+\frac{5}{3}$

$$f(x)=\begin{cases} -x+2, & 0<x \le 2 \\ -\frac{1}{3}x+\frac{5}{3}, & 2<x \le 5 \end{cases}$$

Copyright © 2020 Pearson Education, Inc.

(b) Line through $(-1, 0)$ and $(0, -3)$: $m = \frac{-3-0}{0-(-1)} = -3$, so $y = -3x - 3$

Line through $(0, 3)$ and $(2, -1)$: $m = \frac{-1-3}{2-0} = \frac{-4}{2} = -2$, so $y = -2x + 3$

$$f(x) = \begin{cases} -3x - 3, & -1 < x \le 0 \\ -2x + 3, & 0 < x \le 2 \end{cases}$$

31. (a) Line through $(-1, 1)$ and $(0, 0)$: $y = -x$

Line through $(0, 1)$ and $(1, 1)$: $y = 1$

Line through $(1, 1)$ and $(3, 0)$: $m = \frac{0-1}{3-1} = \frac{-1}{2} = -\frac{1}{2}$, so $y = -\frac{1}{2}(x-1) + 1 = -\frac{1}{2}x + \frac{3}{2}$

$$f(x) = \begin{cases} -x & -1 \le x < 0 \\ 1 & 0 < x \le 1 \\ -\frac{1}{2}x + \frac{3}{2} & 1 < x < 3 \end{cases}$$

(b) Line through $(-2, -1)$ and $(0, 0)$: $y = \frac{1}{2}x$

Line through $(0, 2)$ and $(1, 0)$: $y = -2x + 2$

Line through $(1, -1)$ and $(3, -1)$: $y = -1$

$$f(x) = \begin{cases} \frac{1}{2}x & -2 \le x \le 0 \\ -2x + 2 & 0 < x \le 1 \\ -1 & 1 < x \le 3 \end{cases}$$

32. (a) Line through $\left(\frac{T}{2}, 0\right)$ and $(T, 1)$: $m = \frac{1-0}{T-(T/2)} = \frac{2}{T}$, so $y = \frac{2}{T}\left(x - \frac{T}{2}\right) + 0 = \frac{2}{T}x - 1$

$$f(x) = \begin{cases} 0, & 0 \le x \le \frac{T}{2} \\ \frac{2}{T}x - 1, & \frac{T}{2} < x \le T \end{cases}$$

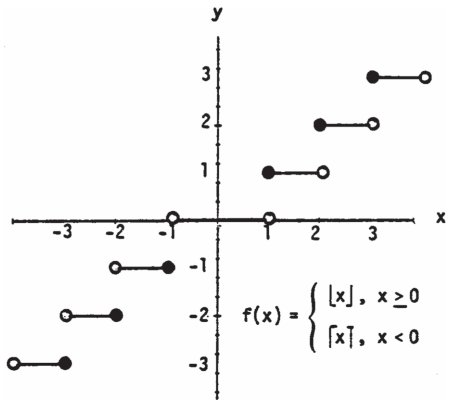
(b) $$f(x) = \begin{cases} A, & 0 \le x < \frac{T}{2} \\ -A, & \frac{T}{2} \le x < T \\ A, & T \le x < \frac{3T}{2} \\ -A, & \frac{3T}{2} \le x \le 2T \end{cases}$$

33. (a) $\lfloor x \rfloor = 0$ for $x \in [0, 1)$ (b) $\lceil x \rceil = 0$ for $x \in (-1, 0]$

34. $\lfloor x \rfloor = \lceil x \rceil$ only when x is an integer.

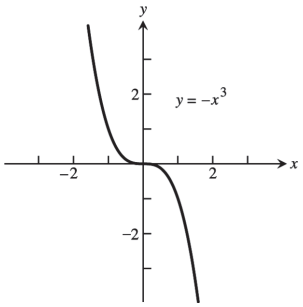
35. For any real number x, $n \le x \le n+1$, where n is an integer. Now: $n \le x \le n+1 \Rightarrow -(n+1) \le -x \le -n$. By definition: $\lceil -x \rceil = -n$ and $\lfloor x \rfloor = n \Rightarrow -\lfloor x \rfloor = -n$. So $\lceil -x \rceil = -\lfloor x \rfloor$ for all real x.

36. To find $f(x)$ you delete the decimal or fractional portion of x, leaving only the integer part.

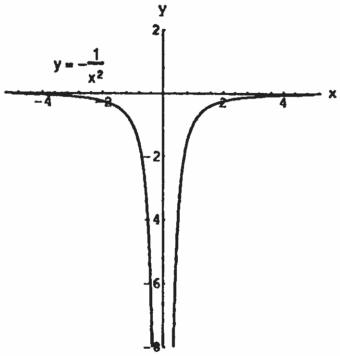


Copyright © 2020 Pearson Education, Inc.

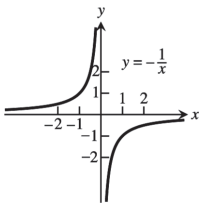
37. Symmetric about the origin
Dec: $-\infty < x < \infty$
Inc: nowhere



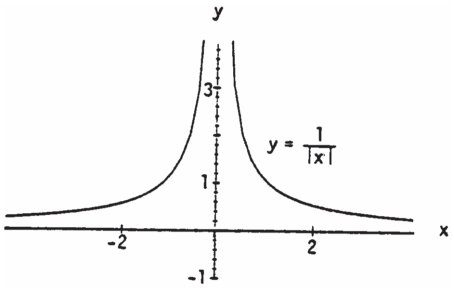
38. Symmetric about the y-axis
Dec: $-\infty < x < 0$
Inc: $0 < x < \infty$



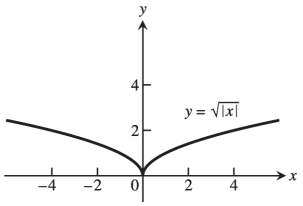
39. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < 0$
$0 < x < \infty$



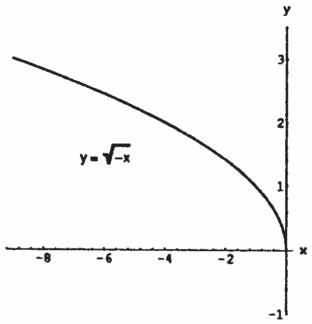
40. Symmetric about the y-axis
Dec: $0 < x < \infty$
Inc: $-\infty < x < 0$



41. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$

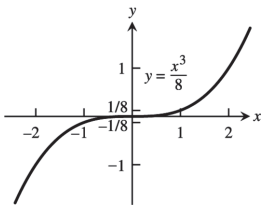


42. No symmetry
Dec: $-\infty < x \le 0$
Inc: nowhere

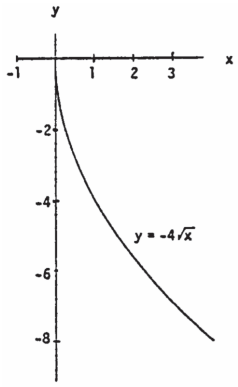


Copyright © 2020 Pearson Education, Inc.

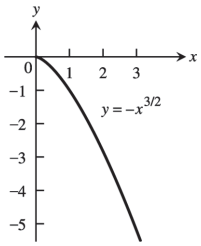
43. Symmetric about the origin
Dec: nowhere
Inc: $-\infty < x < \infty$



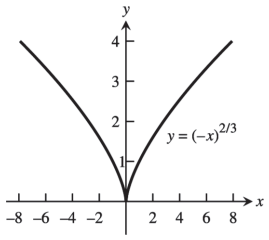
44. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



45. No symmetry
Dec: $0 \le x < \infty$
Inc: nowhere



46. Symmetric about the y-axis
Dec: $-\infty < x \le 0$
Inc: $0 \le x < \infty$



47. Since a horizontal line not through the origin is symmetric with respect to the y-axis, but not with respect to the origin, the function is even.

48. $f(x) = x^{-5} = \frac{1}{x^5}$ and $f(-x) = (-x)^{-5} = \frac{1}{(-x)^5} = -\left(\frac{1}{x^5}\right) = -f(x)$. Thus the function is odd.

49. Since $f(x) = x^2 + 1 = (-x)^2 + 1 = f(-x)$. The function is even.

50. Since $[f(x) = x^2 + x] \ne [f(-x) = (-x)^2 - x]$ and $[f(x) = x^2 + x] \ne [-f(x) = -(x)^2 - x]$ the function is neither even nor odd.

51. Since $g(x) = x^3 + x$, $g(-x) = -x^3 - x = -(x^3 + x) = -g(x)$. So the function is odd.

52. $g(x) = x^4 + 3x^2 - 1 = (-x)^4 + 3(-x)^2 - 1 = g(-x)$, thus the function is even.

53. $g(x) = \frac{1}{x^2 - 1} = \frac{1}{(-x)^2 - 1} = g(-x)$. Thus the function is even.

54. $g(x) = \frac{x}{x^2 - 1}$; $g(-x) = -\frac{x}{x^2 - 1} = -g(x)$. So the function is odd.

55. $h(t) = \frac{1}{t - 1}$; $h(-t) = \frac{1}{-t - 1}$; $-h(t) = \frac{1}{1 - t}$. Since $h(t) \ne -h(t)$ and $h(t) \ne h(-t)$, the function is neither even nor odd.

Copyright © 2020 Pearson Education, Inc.

56. Since $|t^3| = |(-t)^3|, h(t) = h(-t)$ and the function is even.

57. $h(t) = 2t + 1, h(-t) = -2t + 1$. So $h(t) \neq h(-t)$. $-h(t) = -2t - 1$, so $h(t) \neq -h(t)$. The function is neither even nor odd.

58. $h(t) = 2|t| + 1$ and $h(-t) = 2|-t| + 1 = 2|t| + 1$. So $h(t) = h(-t)$ and the function is even.

59. $g(x) = \sin 2x; g(-x) = -\sin 2x = -g(x)$. So the function is odd.

60. $g(x) = \sin x^2; g(-x) = \sin x^2 = g(x)$. So the function is even.

61. $g(x) = \cos 3x; g(-x) = \cos 3x = g(x)$. So the function is even.

62. $g(x) = 1 + \cos x; g(-x) = 1 + \cos x = g(x)$. So the function is even.

63. $s = kt \Rightarrow 25 = k(75) \Rightarrow k = \frac{1}{3} \Rightarrow s = \frac{1}{3}t; 60 = \frac{1}{3}t \Rightarrow t = 180$

64. $K = c\,v^2 \Rightarrow 12960 = c(18)^2 \Rightarrow c = 40 \Rightarrow K = 40v^2; K = 40(10)^2 = 4000$ joules

65. $r = \frac{k}{s} \Rightarrow 6 = \frac{k}{4} \Rightarrow k = 24 \Rightarrow r = \frac{24}{s}; 10 = \frac{24}{s} \Rightarrow s = \frac{12}{5}$

66. $P = \frac{k}{V} \Rightarrow 14.7 = \frac{k}{1000} \Rightarrow k = 14700 \Rightarrow P = \frac{14700}{V}; 23.4 = \frac{14700}{V} \Rightarrow V = \frac{24500}{39} \approx 628.2 \text{ in}^3$

67. $V = f(x) = x(14 - 2x)(22 - 2x) = 4x^3 - 72x^2 + 308x; 0 < x < 7.$

68. (a) Let h = height of the triangle. Since the triangle is isosceles, $\left(\overline{AB}\right)^2 + \left(\overline{AB}\right)^2 = 2^2 \Rightarrow \overline{AB} = \sqrt{2}$. So, $h^2 + 1^2 = \left(\sqrt{2}\right)^2 \Rightarrow h = 1 \Rightarrow B$ is at $(0, 1) \Rightarrow$ slope of $AB = -1 \Rightarrow$ The equation of AB is $y = f(x) = -x + 1; x \in [0, 1]$.
(b) $A(x) = 2xy = 2x(-x + 1) = -2x^2 + 2x; x \in [0, 1]$.

69. (a) Graph h because it is an even function and rises less rapidly than does Graph g.
(b) Graph f because it is an odd function.
(c) Graph g because it is an even function and rises more rapidly than does Graph h.

70. (a) Graph f because it is linear.
(b) Graph g because it contains (0, 1).
(c) Graph h because it is a nonlinear odd function.

Copyright © 2020 Pearson Education, Inc.

71. (a) From the graph, $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow x \in (-2, 0) \cup (4, \infty)$

(b) $\frac{x}{2} > 1 + \frac{4}{x} \Rightarrow \frac{x}{2} - 1 - \frac{4}{x} > 0$

$x > 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} > 0 \Rightarrow \frac{(x-4)(x+2)}{2x} > 0$

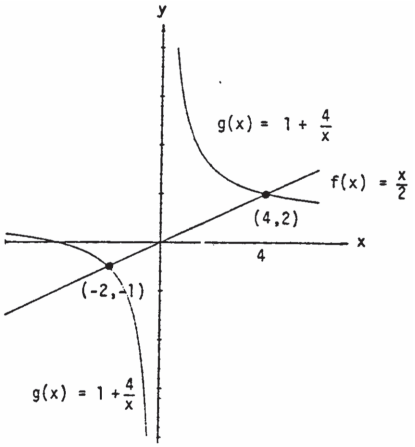
$\Rightarrow x > 4$ since x is positive;

$x < 0$: $\frac{x}{2} - 1 - \frac{4}{x} > 0 \Rightarrow \frac{x^2 - 2x - 8}{2x} < 0 \Rightarrow \frac{(x-4)(x+2)}{2x} < 0$

$\Rightarrow x < -2$ since x is negative;

sign of $(x-4)(x+2)$

Solution interval: $(-2, 0) \cup (4, \infty)$



72. (a) From the graph, $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow x \in (-\infty, -5) \cup (-1, 1)$

(b) Case $x < -1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} > 2$

$\Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$.

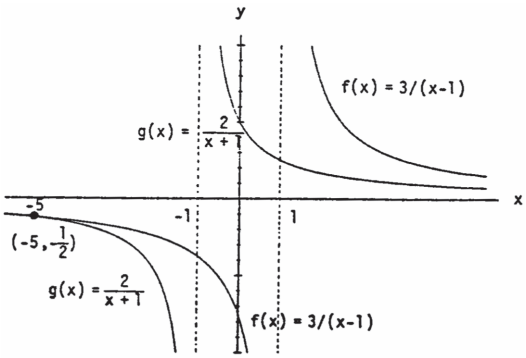
Thus, $x \in (-\infty, -5)$ solves the inequality.

Case $-1 < x < 1$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow \frac{3(x+1)}{x-1} < 2$

$\Rightarrow 3x + 3 > 2x - 2 \Rightarrow x > -5$ which is true if $x > -1$. Thus, $x \in (-1, 1)$ solves the inequality.

Case $1 < x$: $\frac{3}{x-1} < \frac{2}{x+1} \Rightarrow 3x + 3 < 2x - 2 \Rightarrow x < -5$ which is never true if $1 < x$, so no solution here.

In conclusion, $x \in (-\infty, -5) \cup (-1, 1)$.



73. A curve symmetric about the x-axis will not pass the vertical line test because the points (x, y) and $(x, -y)$ lie on the same vertical line. The graph of the function $y = f(x) = 0$ is the x-axis, a horizontal line for which there is a single y-value, 0, for any x.

74. price $= 40 + 5x$, quantity $= 300 - 25x \Rightarrow R(x) = (40 + 5x)(300 - 25x)$

75. $x^2 + x^2 = h^2 \Rightarrow x = \frac{h}{\sqrt{2}} = \frac{\sqrt{2}h}{2}$; cost $= 5(2x) + 10h \Rightarrow C(h) = 10\left(\frac{\sqrt{2}h}{2}\right) + 10h = 5h\left(\sqrt{2} + 2\right)$

76. (a) Note that 2 mi $= 10,560$ ft, so there are $\sqrt{800^2 + x^2}$ feet of river cable at \$180 per foot and $(10,560 - x)$ feet of land cable at \$100 per foot. The cost is $C(x) = 180\sqrt{800^2 + x^2} + 100(10,560 - x)$.

(b) $C(0) = \$1,200,000$

$C(500) \approx \$1,175,812$

$C(1000) \approx \$1,186,512$

$C(1500) \approx \$1,212,000$

$C(2000) \approx \$1,243,732$

$C(2500) \approx \$1,278,479$

$C(3000) \approx \$1,314,870$

Values beyond this are all larger. It would appear that the least expensive location is less than 2000 feet from the point P.

Copyright © 2020 Pearson Education, Inc.

1.2 COMBINING FUNCTIONS; SHIFTING AND SCALING GRAPHS

1. $D_f: -\infty < x < \infty, D_g: x \geq 1 \Rightarrow D_{f+g} = D_{fg}: x \geq 1$

2. $D_f: x+1 \geq 0 \Rightarrow x \geq -1, D_g: x-1 \geq 0 \Rightarrow x \geq 1.$ Therefore $D_{f+g} = D_{fg}: x \geq 1$

3. $D_f: -\infty < x < \infty, D_g: -\infty < x < \infty, D_{f/g}: -\infty < x < \infty, D_{g/f}: -\infty < x < \infty$

4. $D_f: -\infty < x < \infty, D_g: x \geq 0, D_{f/g}: x \geq 0, D_{g/f}: x \geq 0$

5. (a) 2 (b) 22 (c) x^2+2
   (d) $(x+5)^2-3 = x^2+10x+22$ (e) 5 (f) -2
   (g) $x+10$ (h) $(x^2-3)^2-3 = x^4-6x^2+6$

6. (a) $-\frac{1}{3}$ (b) 2 (c) $\frac{1}{x+1}-1 = \frac{-x}{x+1}$
   (d) $\frac{1}{x}$ (e) 0 (f) $\frac{3}{4}$
   (g) $x-2$ (h) $\frac{1}{\frac{1}{x+1}+1} = \frac{1}{\frac{x+2}{x+1}} = \frac{x+1}{x+2}$

7. $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(4-x)) = f(3(4-x)) = f(12-3x) = (12-3x)+1 = 13-3x$

8. $(f\circ g\circ h)(x) = f(g(h(x))) = f(g(x^2)) = f(2(x^2)-1) = f(2x^2-1) = 3(2x^2-1)+4 = 6x^2+1$

9. $(f\circ g\circ h)(x) = f(g(h(x))) = f\left(g\left(\frac{1}{x}\right)\right) = f\left(\frac{1}{\frac{1}{x}+4}\right) = f\left(\frac{x}{1+4x}\right) = \sqrt{\frac{x}{1+4x}+1} = \sqrt{\frac{5x+1}{1+4x}}$

10. $(f\circ g\circ h)(x) = f(g(h(x))) = f\left(g\left(\sqrt{2-x}\right)\right) = f\left(\frac{\left(\sqrt{2-x}\right)^2}{\left(\sqrt{2-x}\right)^2+1}\right) = f\left(\frac{2-x}{3-x}\right) = \frac{\frac{2-x}{3-x}+2}{3-\frac{2-x}{3-x}} = \frac{8-3x}{7-2x}$

11. (a) $(f\circ g)(x)$ (b) $(j\circ g)(x)$ (c) $(g\circ g)(x)$
    (d) $(j\circ j)(x)$ (e) $(g\circ h\circ f)(x)$ (f) $(h\circ j\circ f)(x)$

12. (a) $(f\circ j)(x)$ (b) $(g\circ h)(x)$ (c) $(h\circ h)(x)$
    (d) $(f\circ f)(x)$ (e) $(j\circ g\circ f)(x)$ (f) $(g\circ f\circ h)(x)$

13.

	$g(x)$	$f(x)$	$(f\circ g)(x)$
(a)	$x-7$	$\sqrt{x}$	$\sqrt{x-7}$
(b)	$x+2$	$3x$	$3(x+2) = 3x+6$
(c)	x^2	$\sqrt{x-5}$	$\sqrt{x^2-5}$
(d)	$\frac{x}{x-1}$	$\frac{x}{x-1}$	$\frac{\frac{x}{x-1}}{\frac{x}{x-1}-1} = \frac{x}{x-(x-1)} = x$
(e)	$\frac{1}{x-1}$	$1+\frac{1}{x}$	x
(f)	$\frac{1}{x}$	$\frac{1}{x}$	x

Copyright © 2020 Pearson Education, Inc.

14. (a) $(f\circ g)(x)=|g(x)|=\frac{1}{|x-1|}$.

(b) $(f\circ g)(x)=\frac{g(x)-1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{1}{g(x)}=\frac{x}{x+1}\Rightarrow 1-\frac{x}{x+1}=\frac{1}{g(x)}\Rightarrow \frac{1}{x+1}=\frac{1}{g(x)}$, so $g(x)=x+1$.

(c) Since $(f\circ g)(x)=\sqrt{g(x)}=|x|$, $g(x)=x^2$.

(d) Since $(f\circ g)(x)=f\left(\sqrt{x}\right)=|x|$, $f(x)=x^2$. (Note that the domain of the composition is $[0,\infty)$.)

The completed table is shown. Note that the absolute value sign in part (d) is optional.

$g(x)$	$f(x)$	$(f\circ g)(x)$
$\frac{1}{x-1}$	$\lvert x\rvert$	$\frac{1}{\lvert x-1\rvert}$
$x+1$	$\frac{x-1}{x}$	$\frac{x}{x+1}$
x^2	$\sqrt{x}$	$\lvert x\rvert$
$\sqrt{x}$	x^2	$\lvert x\rvert$

15. (a) $f(g(-1))=f(1)=1$ (b) $g(f(0))=g(-2)=2$ (c) $f(f(-1))=f(0)=-2$
(d) $g(g(2))=g(0)=0$ (e) $g(f(-2))=g(1)=-1$ (f) $f(g(1))=f(-1)=0$

16. (a) $f(g(0))=f(-1)=2-(-1)=3$, where $g(0)=0-1=-1$
(b) $g(f(3))=g(-1)=-(-1)=1$, where $f(3)=2-3=-1$
(c) $g(g(-1))=g(1)=1-1=0$, where $g(-1)=-(-1)=1$
(d) $f(f(2))=f(0)=2-0=2$, where $f(2)=2-2=0$
(e) $g(f(0))=g(2)=2-1=1$, where $f(0)=2-0=2$
(f) $f\left(g\left(\frac{1}{2}\right)\right)=f\left(-\frac{1}{2}\right)=2-\left(-\frac{1}{2}\right)=\frac{5}{2}$, where $g\left(\frac{1}{2}\right)=\frac{1}{2}-1=-\frac{1}{2}$

17. (a) $(f\circ g)(x)=f(g(x))=\sqrt{\frac{1}{x}+1}=\sqrt{\frac{1+x}{x}}$
$(g\circ f)(x)=g(f(x))=\frac{1}{\sqrt{x+1}}$
(b) Domain $(f\circ g)$: $(-\infty,-1]\cup(0,\infty)$, domain $(g\circ f)$: $(-1,\infty)$

18. (a) $(f\circ g)(x)=f(g(x))=1-2\sqrt{x}+x$
$(g\circ f)(x)=g(f(x))=1-|x|$
(b) Domain $(f\circ g)$: $[0,\infty)$, domain $(g\circ f)$: $(-\infty,\infty)$

19. $(f\circ g)(x)=x\Rightarrow f(g(x))=x\Rightarrow \frac{g(x)}{g(x)-2}=x\Rightarrow g(x)=(g(x)-2)x=x\cdot g(x)-2x$
$\Rightarrow g(x)-x\cdot g(x)=-2x\Rightarrow g(x)=-\frac{2x}{1-x}=\frac{2x}{x-1}$

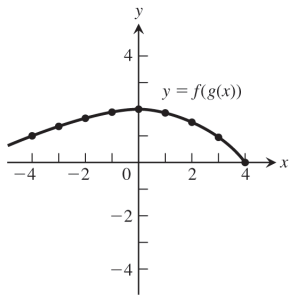
20. $(f\circ g)(x)=x+2\Rightarrow f(g(x))=x+2\Rightarrow 2(g(x))^3-4=x+2\Rightarrow (g(x))^3=\frac{x+6}{2}\Rightarrow g(x)=\sqrt[3]{\frac{x+6}{2}}$

21. $V=V(s)=V(s(t))=V(2t-3)$
$=(2t-3)^2+2(2t-3)+3$
$=4t^2-8t+6$

Copyright © 2020 Pearson Education, Inc.

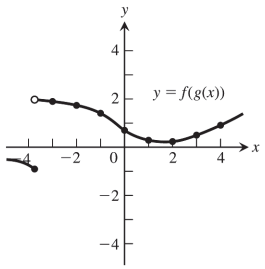
22. (a)

x	–4	–3	–2	–1	0	1	2	3	4
$g(x)$	–2	–1	–0.5	–0.2	0	0.2	0.5	1	2
$f(g(x))$	1	1.3	1.6	1.8	2	1.8	1.5	1	0



(b)

x	–4	–3	–2	–1	0	1	2	3	4
$g(x)$	1.5	0.3	–0.7	–1.5	–2.4	–2.8	–3	–2.7	–2
$f(g(x))$	–0.8	1.9	1.7	1.5	0.7	0.3	0.2	0.5	0.9



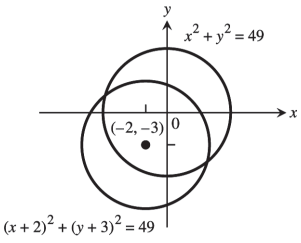
23. (a) $y = -(x+7)^2$ (b) $y = -(x-4)^2$

24. (a) $y = x^2 + 3$ (b) $y = x^2 - 5$

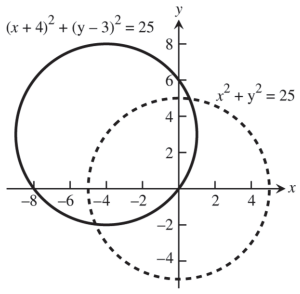
25. (a) Position 4 (b) Position 1 (c) Position 2 (d) Position 3

26. (a) $y = -(x-1)^2 + 4$ (b) $y = -(x+2)^2 + 3$ (c) $y = -(x+4)^2 - 1$ (d) $y = -(x-2)^2$

27.

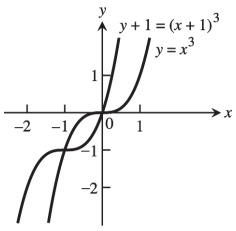


28.

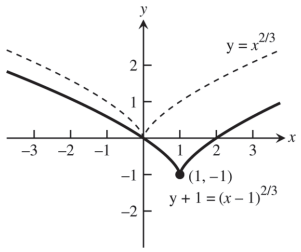


Copyright © 2020 Pearson Education, Inc.

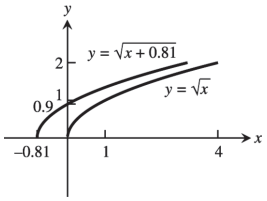
29.



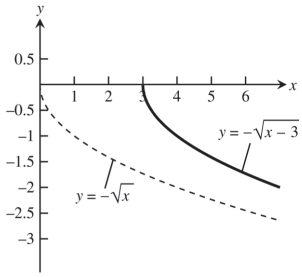
30.



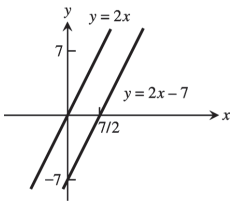
31.



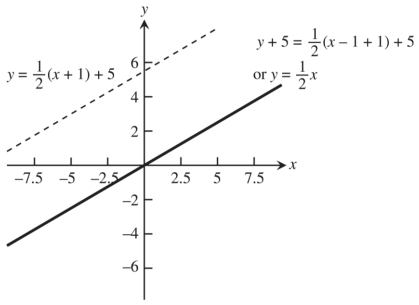
32.



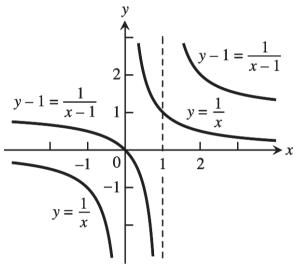
33.



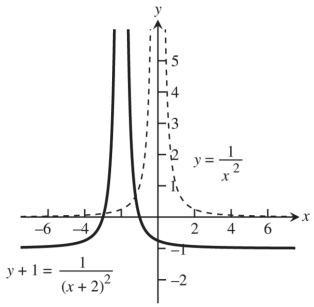
34.



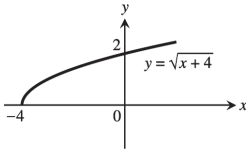
35.



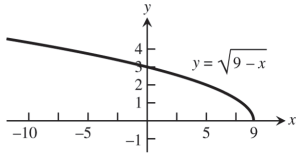
36.



37.

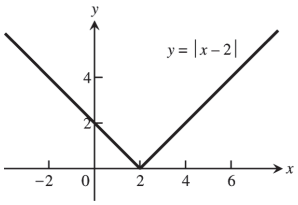


38.

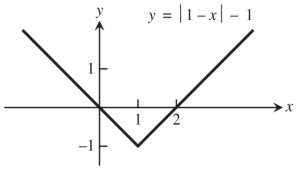


Copyright © 2020 Pearson Education, Inc.

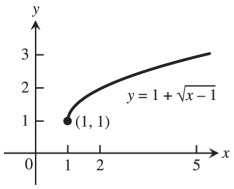
39.



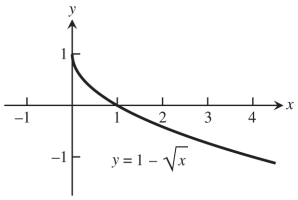
40.



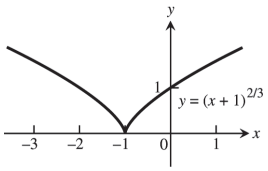
41.



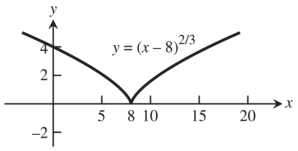
42.



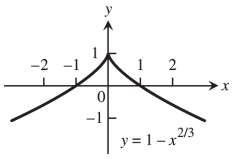
43.



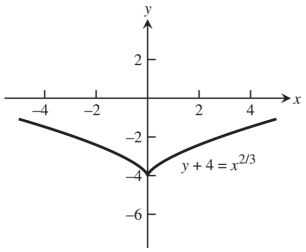
44.



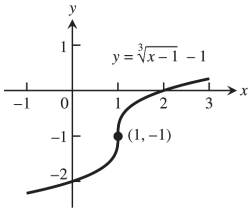
45.



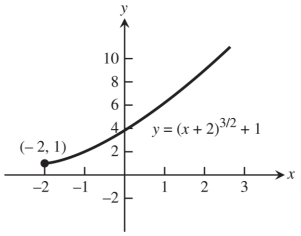
46.



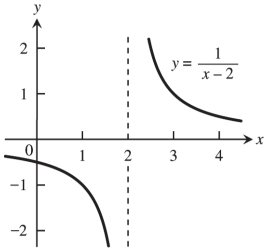
47.



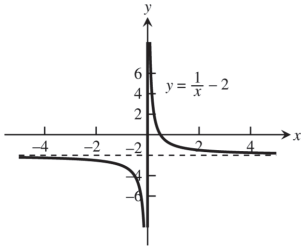
48.



49.

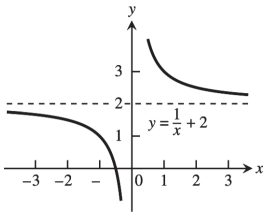


50.

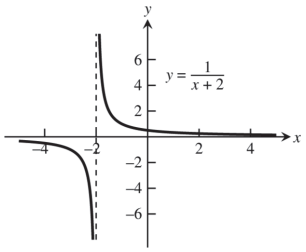


Copyright © 2020 Pearson Education, Inc.

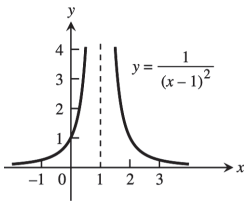
51.



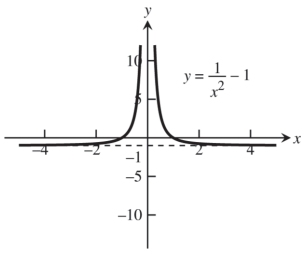
52.



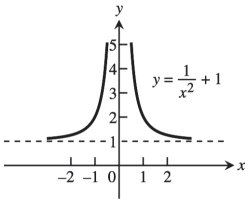
53.



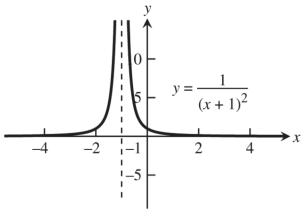
54.



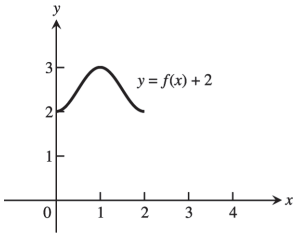
55.



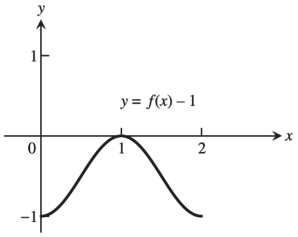
56.



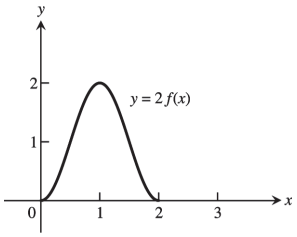
57. (a) domain: [0, 2]; range: [2, 3]



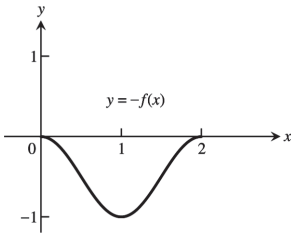
(b) domain: [0, 2]; range: [–1, 0]



(c) domain: [0, 2]; range: [0, 2]

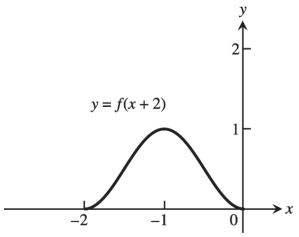


(d) domain: [0, 2]; range: [–1, 0]

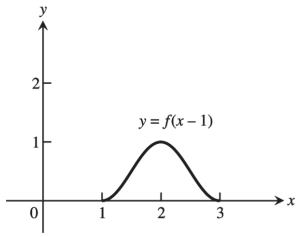


Copyright © 2020 Pearson Education, Inc.

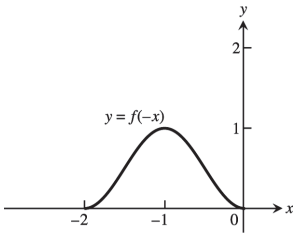
(e) domain: [–2, 0]; range: [0, 1]



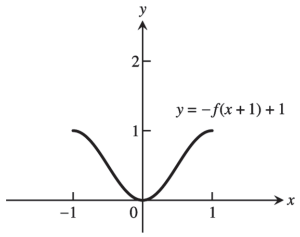
(f) domain: [1, 3]; range: [0,1]



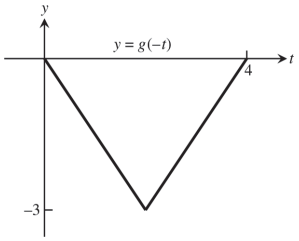
(g) domain: [–2, 0]; range: [0, 1]



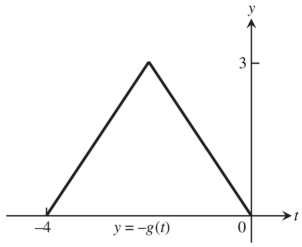
(h) domain: [–1, 1]; range: [0, 1]



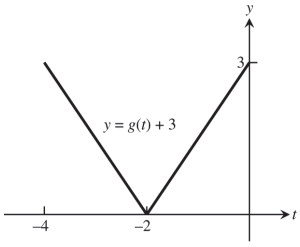
58. (a) domain: [0, 4]; range: [–3, 0]



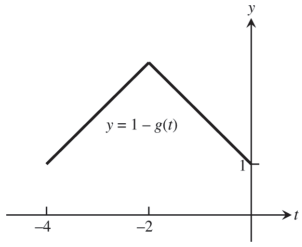
(b) domain: [–4, 0]; range: [0, 3]



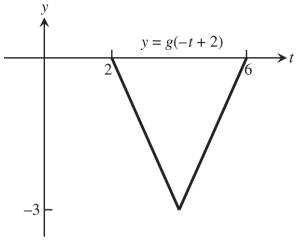
(c) domain: [–4, 0]; range: [0, 3]



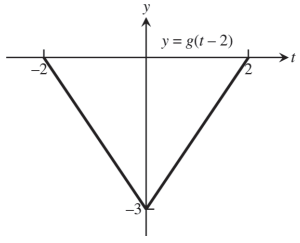
(d) domain: [–4, 0]; range: [1, 4]



(e) domain: [2, 4]; range: [–3, 0]

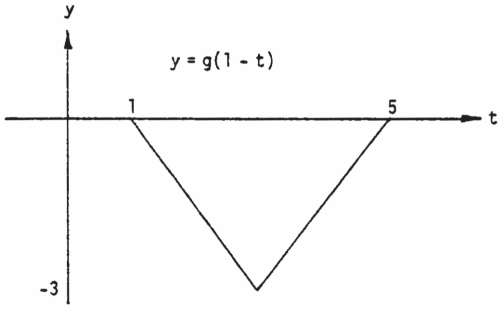


(f) domain: [–2, 2]; range: [–3, 0]

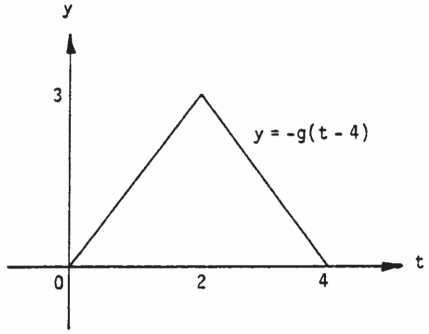


Copyright © 2020 Pearson Education, Inc.

(g) domain: [1, 5]; range: [–3, 0]



(h) domain: [0, 4]; range: [0, 3]



59. $y = 3x^2 - 3$

60. $y = (2x)^2 - 1 = 4x^2 - 1$

61. $y = \frac{1}{2}\left(1 + \frac{1}{x^2}\right) = \frac{1}{2} + \frac{1}{2x^2}$

62. $y = 1 + \frac{1}{(x/3)^2} = 1 + \frac{9}{x^2}$

63. $y = \sqrt{4x+1}$

64. $y = 3\sqrt{x+1}$

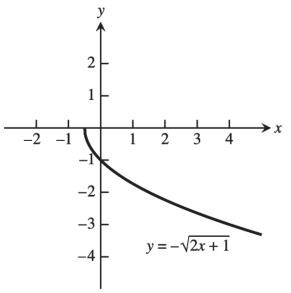
65. $y = \sqrt{4 - \left(\frac{x}{2}\right)^2} = \frac{1}{2}\sqrt{16 - x^2}$

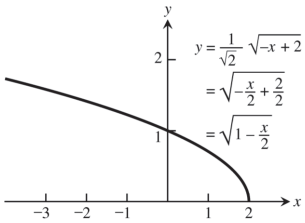
66. $y = \frac{1}{3}\sqrt{4 - x^2}$

67. $y = 1 - (3x)^3 = 1 - 27x^3$

68. $y = 1 - \left(\frac{x}{2}\right)^3 = 1 - \frac{x^3}{8}$

69. Let $y = -\sqrt{2x+1} = f(x)$ and let $g(x) = x^{1/2}$, $h(x) = \left(x + \frac{1}{2}\right)^{1/2}$, $i(x) = \sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}$, and $j(x) = -\left[\sqrt{2}\left(x + \frac{1}{2}\right)^{1/2}\right] = f(x)$. The graph of $h(x)$ is the graph of $g(x)$ shifted left $\frac{1}{2}$ unit; the graph of $i(x)$ is the graph of $h(x)$ stretched vertically by a factor of $\sqrt{2}$; and the graph of $j(x) = f(x)$ is the graph of $i(x)$ reflected across the x-axis.

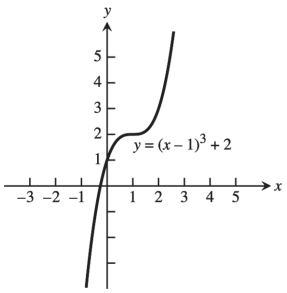




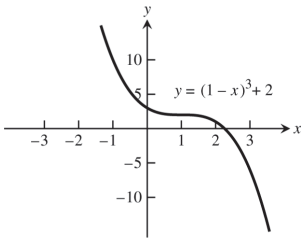
70. Let $y = \sqrt{1 - \frac{x}{2}} = f(x)$. Let $g(x) = (-x)^{1/2}$, $h(x) = (-x+2)^{1/2}$, and $i(x) = \frac{1}{\sqrt{2}}(-x+2)^{1/2} = \sqrt{1 - \frac{x}{2}} = f(x)$. The graph of $g(x)$ is the graph of $y = \sqrt{x}$ reflected across the x-axis. The graph of $h(x)$ is the graph of $g(x)$ shifted right two units. And the graph of $i(x)$ is the graph of $h(x)$ compressed vertically by a factor of $\sqrt{2}$.

Copyright © 2020 Pearson Education, Inc.

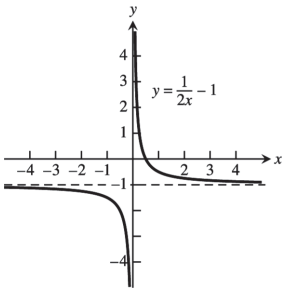
71. $y = f(x) = x^3$. Shift $f(x)$ one unit right followed by a shift two units up to get $g(x) = (x-1)^3 + 2$.



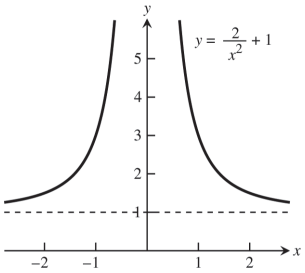
72. $y = (1-x)^3 + 2 = -[(x-1)^3 + (-2)] = f(x)$.
Let $g(x) = x^3$, $h(x) = (x-1)^3$, $i(x) = (x-1)^3 + (-2)$, and $j(x) = -[(x-1)^3 + (-2)]$. The graph of $h(x)$ is the graph of $g(x)$ shifted right one unit; the graph of $i(x)$ is the graph of $h(x)$ shifted down two units; and the graph of $f(x)$ is the graph of $i(x)$ reflected across the x-axis.



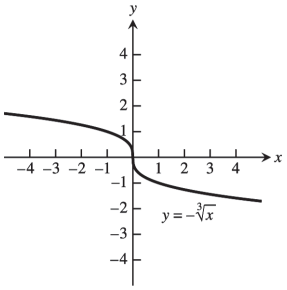
73. Compress the graph of $f(x) = \frac{1}{x}$ horizontally by a factor of 2 to get $g(x) = \frac{1}{2x}$. Then shift $g(x)$ vertically down 1 unit to get $h(x) = \frac{1}{2x} - 1$.



74. Let $f(x) = \frac{1}{x^2}$ and $g(x) = \frac{2}{x^2} + 1 = \frac{1}{\left(\frac{x^2}{2}\right)} + 1$
$= \frac{1}{\left(x/\sqrt{2}\right)^2} + 1 = \frac{1}{\left[\left(1/\sqrt{2}\right)x\right]^2} + 1$. Since $\sqrt{2} \approx 1.4$, we see that the graph of $f(x)$ stretched horizontally by a factor of 1.4 and shifted up 1 unit is the graph of $g(x)$.

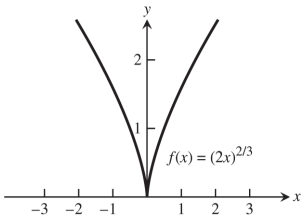


75. Reflect the graph of $y = f(x) = \sqrt[3]{x}$ across the x-axis to get $g(x) = -\sqrt[3]{x}$.

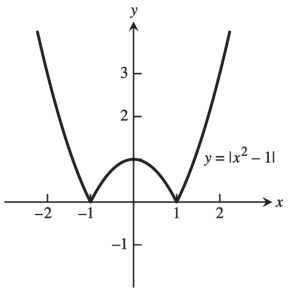


Copyright © 2020 Pearson Education, Inc.

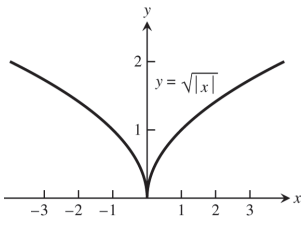
76. $y = f(x) = (-2x)^{2/3} = [(-1)(2)x]^{2/3} = (-1)^{2/3}(2x)^{2/3}$
$= (2x)^{2/3}$. So the graph of $f(x)$ is the graph of $g(x) = x^{2/3}$ compressed horizontally by a factor of 2.



77.



78.



79. (a) $(fg)(-x) = f(-x)g(-x) = f(x)(-g(x)) = -(fg)(x)$, odd

(b) $\left(\frac{f}{g}\right)(-x) = \frac{f(-x)}{g(-x)} = \frac{f(x)}{-g(x)} = -\left(\frac{f}{g}\right)(x)$, odd

(c) $\left(\frac{g}{f}\right)(-x) = \frac{g(-x)}{f(-x)} = \frac{-g(x)}{f(x)} = -\left(\frac{g}{f}\right)(x)$, odd

(d) $f^2(-x) = f(-x)f(-x) = f(x)f(x) = f^2(x)$, even

(e) $g^2(-x) = (g(-x))^2 = (-g(x))^2 = g^2(x)$, even

(f) $(f \circ g)(-x) = f(g(-x)) = f(-g(x)) = f(g(x)) = (f \circ g)(x)$, even

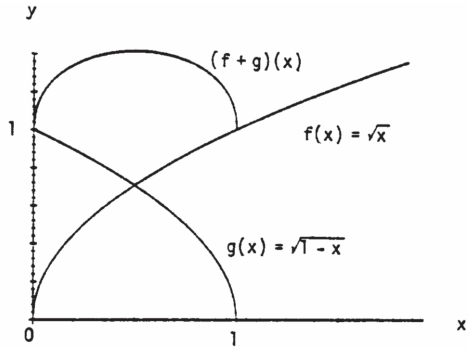
(g) $(g \circ f)(-x) = g(f(-x)) = g(f(x)) = (g \circ f)(x)$, even

(h) $(f \circ f)(-x) = f(f(-x)) = f(f(x)) = (f \circ f)(x)$, even

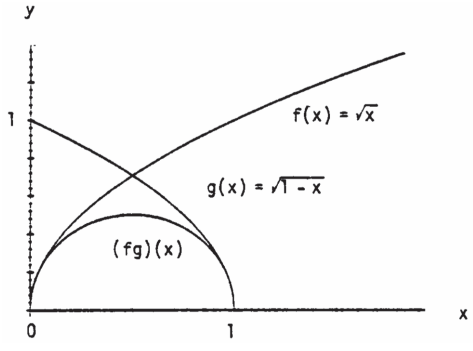
(i) $(g \circ g)(-x) = g(g(-x)) = g(-g(x)) = -g(g(x)) = -(g \circ g)(x)$, odd

80. Yes, $f(x) = 0$ is both even and odd since $f(-x) = 0 = f(x)$ and $f(-x) = 0 = -f(x)$.

81. (a)

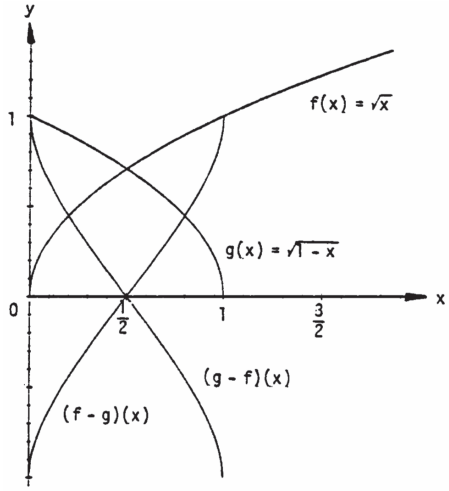


(b)

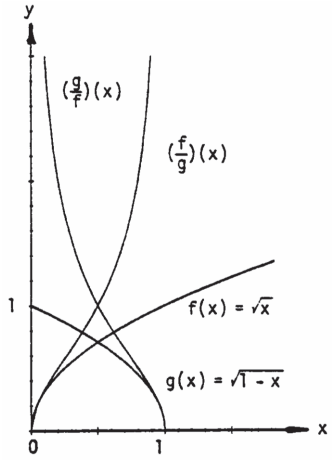


Copyright © 2020 Pearson Education, Inc.

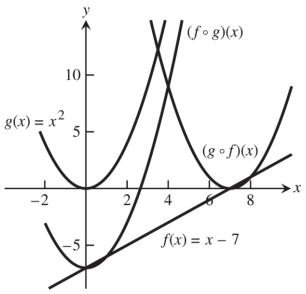
(c)



(d)



82.



1.3 TRIGONOMETRIC FUNCTIONS

1. (a) $s = r\theta = (10)\left(\frac{4\pi}{5}\right) = 8\pi$ m (b) $s = r\theta = (10)(110°)\left(\frac{\pi}{180°}\right) = \frac{110\pi}{18} = \frac{55\pi}{9}$ m

2. $\theta = \frac{s}{r} = \frac{10\pi}{8} = \frac{5\pi}{4}$ radians and $\frac{5\pi}{4}\left(\frac{180°}{\pi}\right) = 225°$

3. $\theta = 80° \Rightarrow \theta = 80°\left(\frac{\pi}{180°}\right) = \frac{4\pi}{9} \Rightarrow s = (6)\left(\frac{4\pi}{9}\right) = 8.4$ in. (since the diameter = 12 in. $\Rightarrow$ radius = 6 in.)

4. $d = 1$ meter $\Rightarrow r = 50$ cm $\Rightarrow \theta = \frac{s}{r} = \frac{30}{50} = 0.6$ rad or $0.6\left(\frac{180°}{\pi}\right) \approx 34°$

5.

θ	$-\pi$	$-\frac{2\pi}{3}$	0	$\frac{\pi}{2}$	$\frac{3\pi}{4}$
$\sin\theta$	0	$-\frac{\sqrt{3}}{2}$	0	1	$\frac{1}{\sqrt{2}}$
$\cos\theta$	-1	$-\frac{1}{2}$	1	0	$-\frac{1}{\sqrt{2}}$
$\tan\theta$	0	$\sqrt{3}$	0	und.	-1
$\cot\theta$	und.	$\frac{1}{\sqrt{3}}$	und.	0	-1
$\sec\theta$	-1	-2	1	und.	$-\sqrt{2}$
$\csc\theta$	und.	$-\frac{2}{\sqrt{3}}$	und.	1	$\sqrt{2}$

6.

θ	$-\frac{3\pi}{2}$	$-\frac{\pi}{3}$	$-\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{5\pi}{6}$
$\sin\theta$	1	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\cos\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$
$\tan\theta$	und.	$-\sqrt{3}$	$-\frac{1}{\sqrt{3}}$	1	$-\frac{1}{\sqrt{3}}$
$\cot\theta$	0	$-\frac{1}{\sqrt{3}}$	$-\sqrt{3}$	1	$-\sqrt{3}$
$\sec\theta$	und.	2	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	$-\frac{2}{\sqrt{3}}$
$\csc\theta$	1	$-\frac{2}{\sqrt{3}}$	-2	$\sqrt{2}$	2

Copyright © 2020 Pearson Education, Inc.

7. $\cos x = -\frac{4}{5}, \tan x = -\frac{3}{4}$

8. $\sin x = \frac{2}{\sqrt{5}}, \cos x = \frac{1}{\sqrt{5}}$

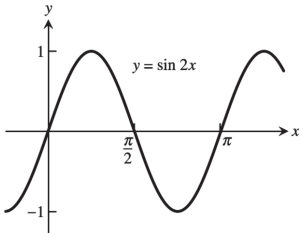
9. $\sin x = -\frac{\sqrt{8}}{3}, \tan x = -\sqrt{8}$

10. $\sin x = \frac{12}{13}, \tan x = -\frac{12}{5}$

11. $\sin x = -\frac{1}{\sqrt{5}}, \cos x = -\frac{2}{\sqrt{5}}$

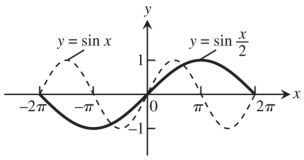
12. $\cos x = -\frac{\sqrt{3}}{2}, \tan x = \frac{1}{\sqrt{3}}$

13.



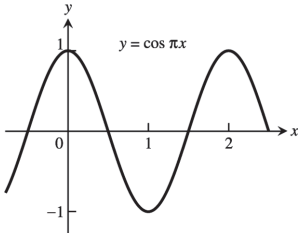
period = π

14.



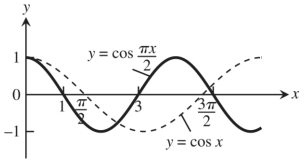
period = 4π

15.



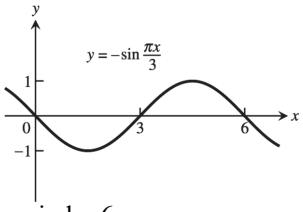
period = 2

16.



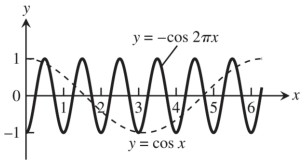
period = 4

17.



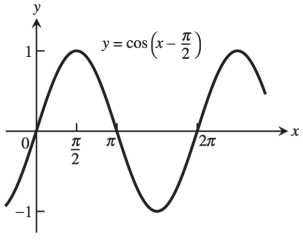
period = 6

18.



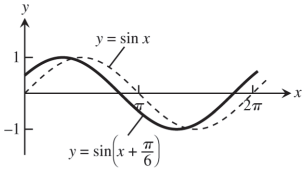
period = 1

19.



period = 2π

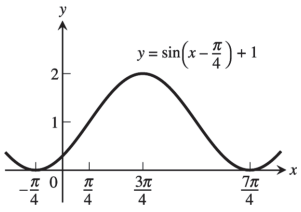
20.



period = 2π

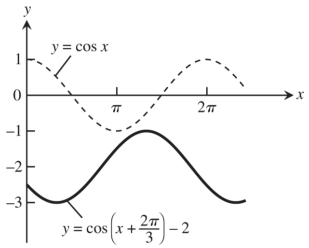
Copyright © 2020 Pearson Education, Inc.

21.



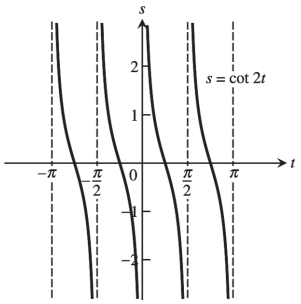
period = 2π

22.

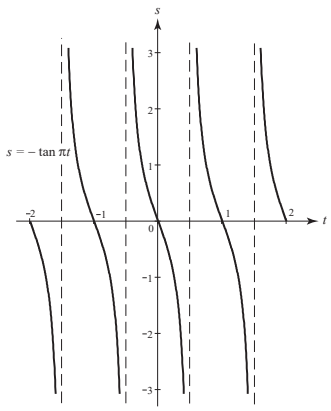


period = 2π

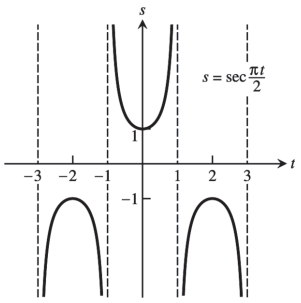
23. period = $\frac{\pi}{2}$, symmetric about the origin



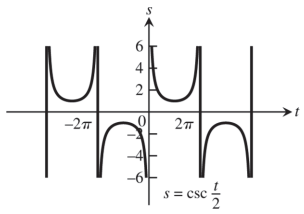
24. period = 1, symmetric about the origin



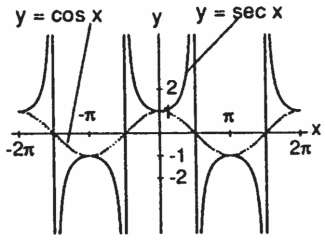
25. period = 4, symmetric about the *s*-axis



26. period = 4π, symmetric about the origin

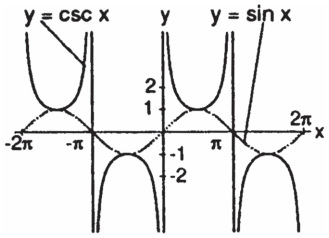


27. (a) Cos x and sec x are positive for x in the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$; and cos x and sec x are negative for x in the intervals $\left(-\frac{3\pi}{2}, -\frac{\pi}{2}\right)$ and $\left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$. Sec x is undefined when cos x is 0. The range of sec x is $(-\infty, -1] \cup [1, \infty)$; the range of cos x is $[-1, 1]$.

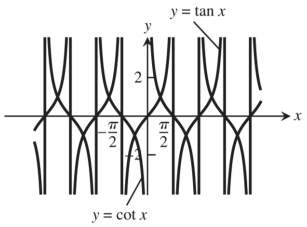


Copyright © 2020 Pearson Education, Inc.

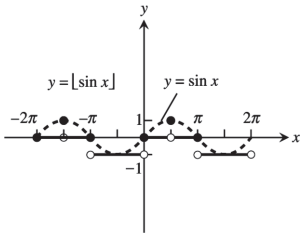
(b) Sin x and csc x are positive for x in the intervals $\left(-\frac{3\pi}{2}, -\pi\right)$ and $(0, \pi)$; and sin x and csc x are negative for x in the intervals $(-\pi, 0)$ and $\left(\pi, \frac{3\pi}{2}\right)$. Csc x is undefined when sin x is 0. The range of csc x is $(-\infty, -1] \cup [1, \infty)$; the range of sin x is $[-1, 1]$.



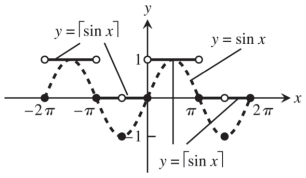
28. Since $\cot x = \frac{1}{\tan x}$, cot x is undefined when $\tan x = 0$ and is zero when tan x is undefined. As tan x approaches zero through positive values, cot x approaches infinity. Also, cot x approaches negative infinity as tan x approaches zero through negative values.



29. D: $-\infty < x < \infty$; R: $y = -1, 0, 1$



30. D: $-\infty < x < \infty$; R: $y = -1, 0, 1$



31. $\cos\left(x - \frac{\pi}{2}\right) = \cos x \cos\left(-\frac{\pi}{2}\right) - \sin x \sin\left(-\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(-1) = \sin x$

32. $\cos\left(x + \frac{\pi}{2}\right) = \cos x \cos\left(\frac{\pi}{2}\right) - \sin x \sin\left(\frac{\pi}{2}\right) = (\cos x)(0) - (\sin x)(1) = -\sin x$

33. $\sin\left(x + \frac{\pi}{2}\right) = \sin x \cos\left(\frac{\pi}{2}\right) + \cos x \sin\left(\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(1) = \cos x$

34. $\sin\left(x - \frac{\pi}{2}\right) = \sin x \cos\left(-\frac{\pi}{2}\right) + \cos x \sin\left(-\frac{\pi}{2}\right) = (\sin x)(0) + (\cos x)(-1) = -\cos x$

35. $\cos(A - B) = \cos(A + (-B)) = \cos A \cos(-B) - \sin A \sin(-B) = \cos A \cos B - \sin A(-\sin B)$
$= \cos A \cos B + \sin A \sin B$

36. $\sin(A - B) = \sin(A + (-B)) = \sin A \cos(-B) + \cos A \sin(-B) = \sin A \cos B + \cos A(-\sin B)$
$= \sin A \cos B - \cos A \sin B$

37. If $B = A$, $A - B = 0 \Rightarrow \cos(A - B) = \cos 0 = 1$. Also $\cos(A - B) = \cos(A - A) = \cos A \cos A + \sin A \sin A$ $= \cos^2 A + \sin^2 A$. Therefore, $\cos^2 A + \sin^2 A = 1$.

38. If $B = 2\pi$, then $\cos(A + 2\pi) = \cos A \cos 2\pi - \sin A \sin 2\pi = (\cos A)(1) - (\sin A)(0) = \cos A$ and $\sin(A + 2\pi) = \sin A \cos 2\pi + \cos A \sin 2\pi = (\sin A)(1) + (\cos A)(0) = \sin A$. The result agrees with the fact that the cosine and sine functions have period 2π.

39. $\cos(\pi + x) = \cos \pi \cos x - \sin \pi \sin x = (-1)(\cos x) - (0)(\sin x) = -\cos x$

Copyright © 2020 Pearson Education, Inc.

40. $\sin(2\pi - x) = \sin 2\pi\cos(-x) + \cos(2\pi)\sin(-x) = (0)(\cos(-x)) + (1)(\sin(-x)) = -\sin x$

41. $\sin\left(\frac{3\pi}{2} - x\right) = \sin\left(\frac{3\pi}{2}\right)\cos(-x) + \cos\left(\frac{3\pi}{2}\right)\sin(-x) = (-1)(\cos x) + (0)(\sin(-x)) = -\cos x$

42. $\cos\left(\frac{3\pi}{2} + x\right) = \cos\left(\frac{3\pi}{2}\right)\cos x - \sin\left(\frac{3\pi}{2}\right)\sin x = (0)(\cos x) - (-1)(\sin x) = \sin x$

43. $\sin\frac{7\pi}{12} = \sin\left(\frac{\pi}{4} + \frac{\pi}{3}\right) = \sin\frac{\pi}{4}\cos\frac{\pi}{3} + \cos\frac{\pi}{4}\sin\frac{\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(\frac{1}{2}\right) + \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = \frac{\sqrt{6} + \sqrt{2}}{4}$

44. $\cos\frac{11\pi}{12} = \cos\left(\frac{\pi}{4} + \frac{2\pi}{3}\right) = \cos\frac{\pi}{4}\cos\frac{2\pi}{3} - \sin\frac{\pi}{4}\sin\frac{2\pi}{3} = \left(\frac{\sqrt{2}}{2}\right)\left(-\frac{1}{2}\right) - \left(\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{3}}{2}\right) = -\frac{\sqrt{2} + \sqrt{6}}{4}$

45. $\cos\frac{\pi}{12} = \cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\left(-\frac{\pi}{4}\right) - \sin\frac{\pi}{3}\sin\left(-\frac{\pi}{4}\right) = \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) - \left(\frac{\sqrt{3}}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

46. $\sin\frac{5\pi}{12} = \sin\left(\frac{2\pi}{3} - \frac{\pi}{4}\right) = \sin\left(\frac{2\pi}{3}\right)\cos\left(-\frac{\pi}{4}\right) + \cos\left(\frac{2\pi}{3}\right)\sin\left(-\frac{\pi}{4}\right) = \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(-\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right) = \frac{1+\sqrt{3}}{2\sqrt{2}}$

47. $\cos^2\frac{\pi}{8} = \frac{1 + \cos\left(\frac{2\pi}{8}\right)}{2} = \frac{1 + \frac{\sqrt{2}}{2}}{2} = \frac{2 + \sqrt{2}}{4}$

48. $\cos^2\frac{5\pi}{12} = \frac{1 + \cos\left(\frac{10\pi}{12}\right)}{2} = \frac{1 + \left(-\frac{\sqrt{3}}{2}\right)}{2} = \frac{2 - \sqrt{3}}{4}$

49. $\sin^2\frac{\pi}{12} = \frac{1 - \cos\left(\frac{2\pi}{12}\right)}{2} = \frac{1 - \frac{\sqrt{3}}{2}}{2} = \frac{2 - \sqrt{3}}{4}$

50. $\sin^2\frac{3\pi}{8} = \frac{1 - \cos\left(\frac{6\pi}{8}\right)}{2} = \frac{1 - \left(-\frac{\sqrt{2}}{2}\right)}{2} = \frac{2 + \sqrt{2}}{4}$

51. $\sin^2\theta = \frac{3}{4} \Rightarrow \sin\theta = \pm\frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}, \frac{2\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

52. $\sin^2\theta = \cos^2\theta \Rightarrow \frac{\sin^2\theta}{\cos^2\theta} = \frac{\cos^2\theta}{\cos^2\theta} \Rightarrow \tan^2\theta = 1 \Rightarrow \tan\theta = \pm 1 \Rightarrow \theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

53. $\sin 2\theta - \cos\theta = 0 \Rightarrow 2\sin\theta\cos\theta - \cos\theta = 0 \Rightarrow \cos\theta(2\sin\theta - 1) = 0 \Rightarrow \cos\theta = 0$ or $2\sin\theta - 1 = 0$
$\Rightarrow \cos\theta = 0$ or $\sin\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{2}, \frac{3\pi}{2}$, or $\theta = \frac{\pi}{6}, \frac{5\pi}{6} \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

54. $\cos 2\theta + \cos\theta = 0 \Rightarrow 2\cos^2\theta - 1 + \cos\theta = 0 \Rightarrow 2\cos^2\theta + \cos\theta - 1 = 0 \Rightarrow (\cos\theta + 1)(2\cos\theta - 1) = 0$
$\Rightarrow \cos\theta + 1 = 0$ or $2\cos\theta - 1 = 0 \Rightarrow \cos\theta = -1$ or $\cos\theta = \frac{1}{2} \Rightarrow \theta = \pi$ or $\theta = \frac{\pi}{3}, \frac{5\pi}{3} \Rightarrow \theta = \frac{\pi}{3}, \pi, \frac{5\pi}{3}$

55. $\tan(A+B) = \frac{\sin(A+B)}{\cos(A+B)} = \frac{\sin A\cos B + \cos A\cos B}{\cos A\cos B - \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} + \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} - \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

56. $\tan(A-B) = \frac{\sin(A-B)}{\cos(A-B)} = \frac{\sin A\cos B - \cos A\cos B}{\cos A\cos B + \sin A\sin B} = \frac{\frac{\sin A\cos B}{\cos A\cos B} - \frac{\cos A\sin B}{\cos A\cos B}}{\frac{\cos A\cos B}{\cos A\cos B} + \frac{\sin A\sin B}{\cos A\cos B}} = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

57. According to the figure in the text, we have the following: By the law of cosines, $c^2 = a^2 + b^2 - 2ab\cos\theta$
$= 1^2 + 1^2 - 2\cos(A-B) = 2 - 2\cos(A-B)$. By distance formula, $c^2 = (\cos A - \cos B)^2 + (\sin A - \sin B)^2$
$= \cos^2 A - 2\cos A\cos B + \cos^2 B + \sin^2 A - 2\sin A\sin B + \sin^2 B = 2 - 2(\cos A\cos B + \sin A\sin B)$. Thus
$c^2 = 2 - 2\cos(A-B) = 2 - 2(\cos A\cos B + \sin A\sin B) \Rightarrow \cos(A-B) = \cos A\cos B + \sin A\sin B$.

Copyright © 2020 Pearson Education, Inc.

58. (a) $\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\sin\theta = \cos\left(\frac{\pi}{2}-\theta\right)$ and $\cos\theta = \sin\left(\frac{\pi}{2}-\theta\right)$
Let $\theta = A+B$
$\sin(A+B) = \cos\left[\frac{\pi}{2}-(A+B)\right] = \cos\left[\left(\frac{\pi}{2}-A\right)-B\right] = \cos\left(\frac{\pi}{2}-A\right)\cos B + \sin\left(\frac{\pi}{2}-A\right)\sin B$
$= \sin A\cos B + \cos A\sin B$

(b) $\cos(A-B) = \cos A\cos B + \sin A\sin B$
$\cos(A-(-B)) = \cos A\cos(-B) + \sin A\sin(-B)$
$\Rightarrow \cos(A+B) = \cos A\cos(-B) + \sin A\sin(-B) = \cos A\cos B + \sin A(-\sin B) = \cos A\cos B - \sin A\sin B$
Because the cosine function is even and the sine functions is odd.

59. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(60°) = 4 + 9 - 12\cos(60°) = 13 - 12\left(\frac{1}{2}\right) = 7.$
Thus, $c = \sqrt{7} \approx 2.65$.

60. $c^2 = a^2 + b^2 - 2ab\cos C = 2^2 + 3^2 - 2(2)(3)\cos(40°) = 13 - 12\cos(40°)$. Thus, $c = \sqrt{13 - 12\cos 40°} \approx 1.951$.

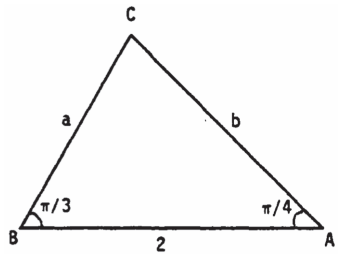
61. From the figures in the text, we see that $\sin B = \frac{h}{c}$. If C is an acute angle, then $\sin C = \frac{h}{b}$. On the other hand, if C is obtuse (as in the figure on the right in the text), then $\sin C = \sin(\pi - C) = \frac{h}{b}$. Thus, in either case, $h = b\sin C = c\sin B \Rightarrow ah = ab\sin C = ac\sin B$.

By the law of cosines, $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$ and $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$. Moreover, since the sum of the interior angles of triangle is π, we have $\sin A = \sin(\pi - (B+C)) = \sin(B+C) = \sin B\cos C + \cos B\sin C$
$= \left(\frac{h}{c}\right)\left[\frac{a^2 + b^2 - c^2}{2ab}\right] + \left[\frac{a^2 + c^2 - b^2}{2ac}\right]\left(\frac{h}{b}\right) = \left(\frac{h}{2abc}\right)(2a^2 + b^2 - c^2 + c^2 - b^2) = \frac{ah}{bc} \Rightarrow ah = bc\sin A.$

Combining our results we have $ah = ab\sin C$, $ah = ac\sin B$, and $ah = bc\sin A$. Dividing by abc gives $\frac{h}{bc} = \underbrace{\frac{\sin A}{a} = \frac{\sin C}{c} = \frac{\sin B}{b}}_{\text{law of sines}}$.

62. By the law of sines, $\frac{\sin A}{2} = \frac{\sin B}{3} = \frac{\sqrt{3}/2}{c}$. By Exercise 59 we know that $c = \sqrt{7}$. Thus $\sin B = \frac{3\sqrt{3}}{2\sqrt{7}} \simeq 0.982$.

63. From the figure at the right and the law of cosines,
$b^2 = a^2 + 2^2 - 2(2a)\cos B$
$= a^2 + 4 - 4a\left(\frac{1}{2}\right) = a^2 - 2a + 4.$



Applying the law of sines to the figure, $\frac{\sin A}{a} = \frac{\sin B}{b}$
$\Rightarrow \frac{\sqrt{2}/2}{a} = \frac{\sqrt{3}/2}{b} \Rightarrow b = \sqrt{\frac{3}{2}}\,a$. Thus, combining results,
$a^2 - 2a + 4 = b^2 = \frac{3}{2}a^2 \Rightarrow 0 = \frac{1}{2}a^2 + 2a - 4 \Rightarrow 0 = a^2 + 4a - 8$. From the quadratic formula and the fact that $a > 0$, we have $a = \frac{-4 + \sqrt{4^2 - 4(1)(-8)}}{2} = \frac{4\sqrt{3} - 4}{2} \simeq 1.464$.

Copyright © 2020 Pearson Education, Inc.

64. $\tan\gamma = \frac{h}{c} \Rightarrow c = \frac{h}{\tan\gamma}$

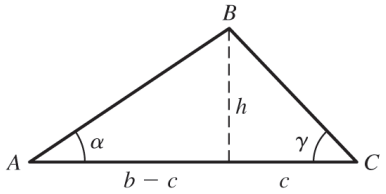
$\tan\alpha = \frac{h}{b-c} = \frac{h}{b-\frac{h}{\tan\gamma}} = \frac{h\tan\gamma}{b\tan\gamma - h} \Rightarrow$

$b\tan\alpha\tan\gamma - h\tan\alpha = h\tan\gamma \Rightarrow$

$b\tan\alpha\tan\gamma = h\tan\alpha + h\tan\gamma \Rightarrow$

$b\tan\alpha\tan\gamma = h(\tan\alpha + \tan\gamma) \Rightarrow$

$h = \frac{b\tan\alpha\tan\gamma}{\tan\alpha + \tan\gamma}$

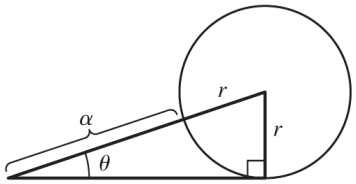


65. $\sin\theta = \frac{r}{\alpha + r}$

$\Rightarrow \alpha\sin\theta + r\sin\theta = r$

$\Rightarrow \alpha\sin\theta = r - r\sin\theta = r(1-\sin\theta)$

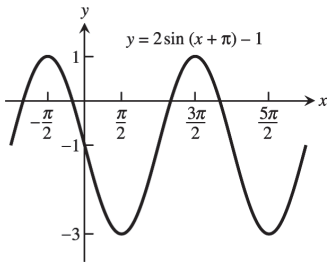
$\Rightarrow r = \frac{\alpha\sin\theta}{1-\sin\theta}$



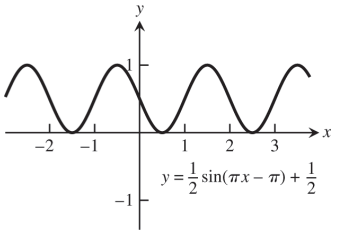
66. (a) The graphs of $y = \sin x$ and $y = x$ nearly coincide when x is near the origin (when the calculator is in radians mode).

(b) In degree mode, when x is near zero degrees the sine of x is much closer to zero than x itself. The curves look like intersecting straight lines near the origin when the calculator is in degree mode.

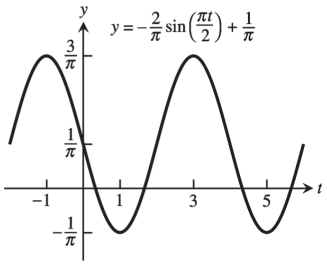
67. $A = 2, B = 2\pi, C = -\pi, D = -1$



68. $A = \frac{1}{2}, B = 2, C = 1, D = \frac{1}{2}$

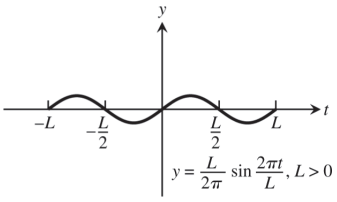


69. $A = -\frac{2}{\pi}, B = 4, C = 0, D = \frac{1}{\pi}$



Copyright © 2020 Pearson Education, Inc.

70. $A = \frac{L}{2\pi}, B = L, C = 0, D = 0$



71–74. Example CAS commands:

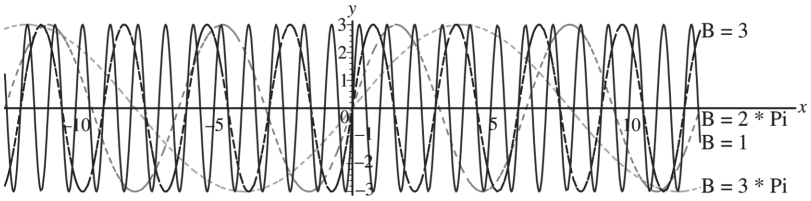
Maple:

```
f :=x -> A*sin((2*Pi/B)*(x-C))+D1;
A:=3; C:=0; D1:=0;
f_list := [seq(f(x), B=[1,3,2*Pi,5*Pi])];
plot(f_list, x=-4*Pi..4*Pi, scaling=constrained,
     color=[red,blue,green,cyan], linestyle=[1,3,4,7],
     legend=["B=1", "B=3","B=2*Pi","B=3*Pi"],
     title="#71 (Section 1.3)");
```

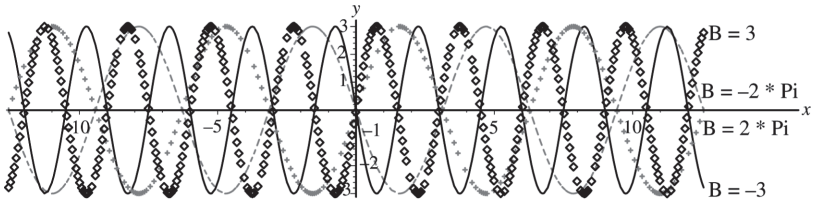
Mathematica:

```
Clear[a, b, c, d, f, x]
f[x_]:=a Sin[2π/b (x − c)] + d
Plot[f[x]/.{a → 3, b → 1, c → 0, d → 0}, {x, −4π, 4π}]
```

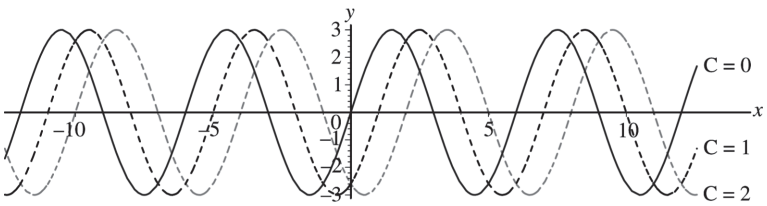
71. (a) The graph stretches horizontally.



(b) The period remains the same: period = $|B|$. The graph has a horizontal shift of $\frac{1}{2}$ period.



72. (a) The graph is shifted right C units.

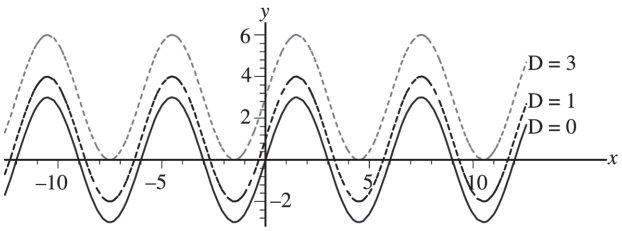


(b) The graph is shifted left C units.

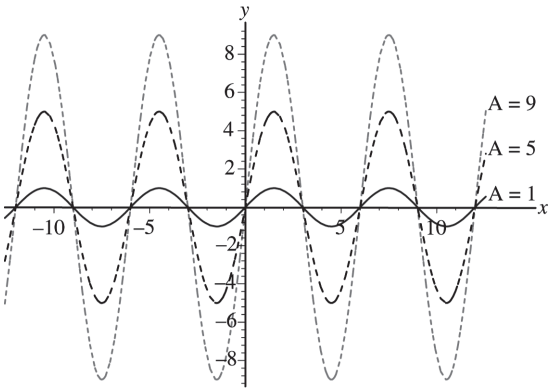
(c) A shift of ± one period will produce no apparent shift. $|C| = 6$

Copyright © 2020 Pearson Education, Inc.

73. (a) The graph shifts upwards $|D|$ units for $D > 0$
(b) The graph shifts down $|D|$ units for $D < 0$.



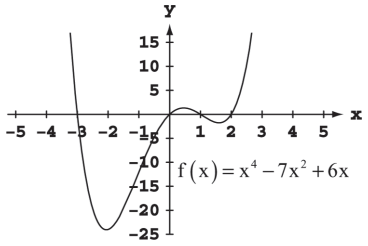
74. (a) The graph stretches $|A|$ units. (b) For $A < 0$, the graph is inverted.



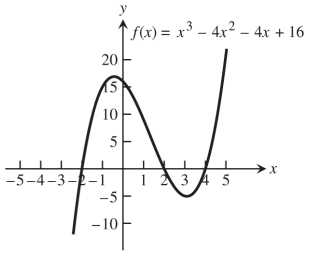
1.4 GRAPHING WITH SOFTWARE

1–4. The most appropriate viewing window displays the maxima, minima, intercepts, and end behavior of the graphs and has little unused space.

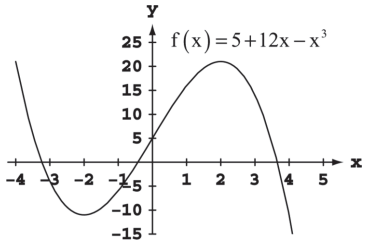
1. d.



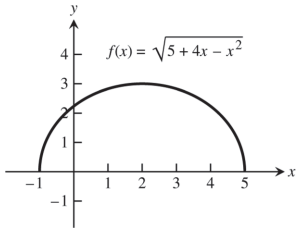
2. c.



3. d.



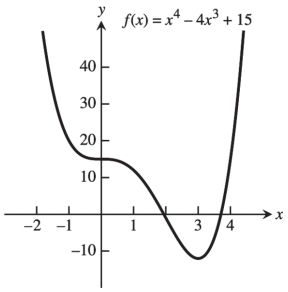
4. b.



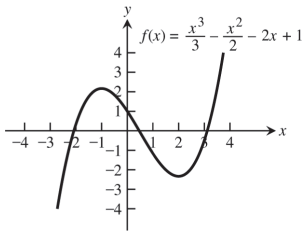
Copyright © 2020 Pearson Education, Inc.

5–30. For any display there are many appropriate display widows. The graphs given as answers in Exercises 5–30 are not unique in appearance.

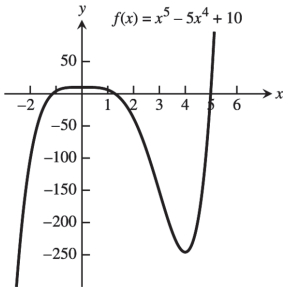
5. $[-2, 5]$ by $[-15, 40]$



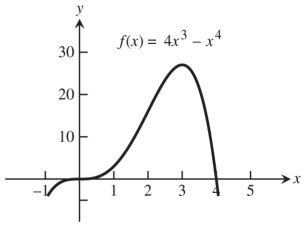
6. $[-4, 4]$ by $[-4, 4]$



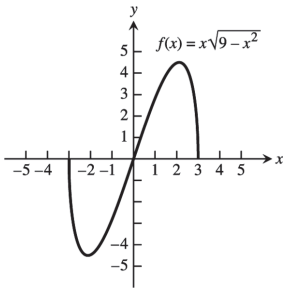
7. $[-2, 6]$ by $[-250, 50]$



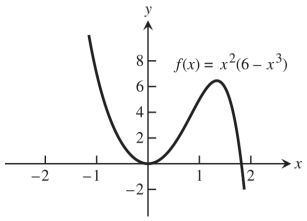
8. $[-1, 5]$ by $[-5, 30]$



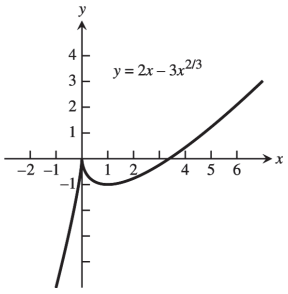
9. $[-4, 4]$ by $[-5, 5]$



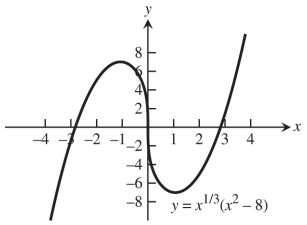
10. $[-2, 2]$ by $[-2, 8]$



11. $[-2, 6]$ by $[-5, 4]$

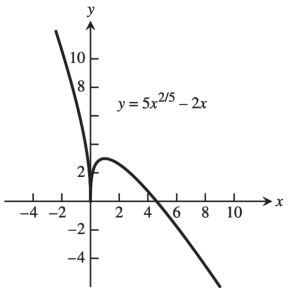


12. $[-4, 4]$ by $[-8, 8]$

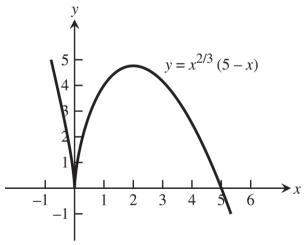


Copyright © 2020 Pearson Education, Inc.

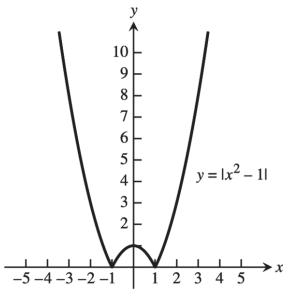
13. $[-1, 6]$ by $[-1, 4]$



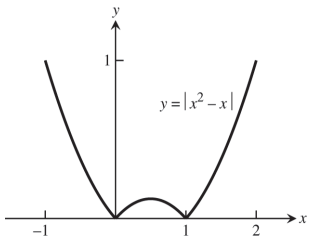
14. $[-1, 6]$ by $[-1, 5]$



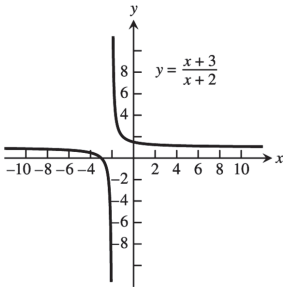
15. $[-3, 3]$ by $[0, 10]$



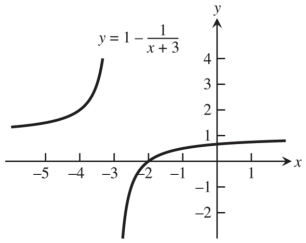
16. $[-1, 2]$ by $[0, 1]$



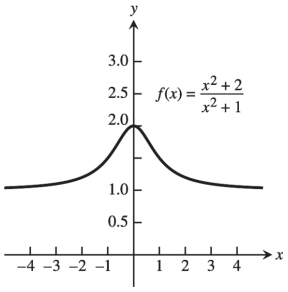
17. $[-5, 1]$ by $[-5, 5]$



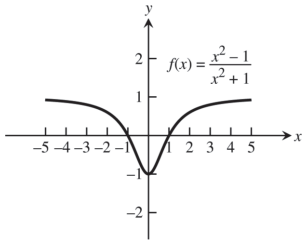
18. $[-5, 1]$ by $[-2, 4]$



19. $[-4, 4]$ by $[0, 3]$

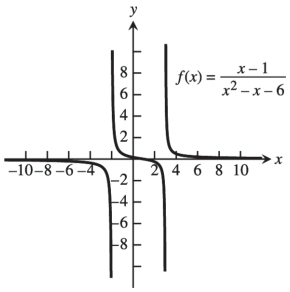


20. $[-5, 5]$ by $[-2, 2]$

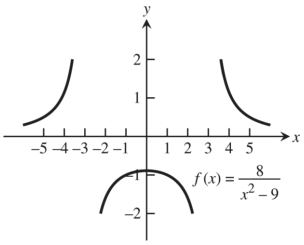


Copyright © 2020 Pearson Education, Inc.

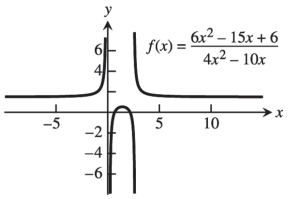
21. [−10, 10] by [−6, 6]



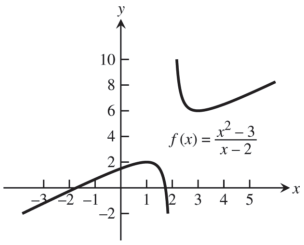
22. [−5, 5] by [−2, 2]



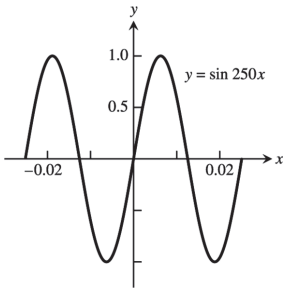
23. [−6, 10] by [−6, 6]



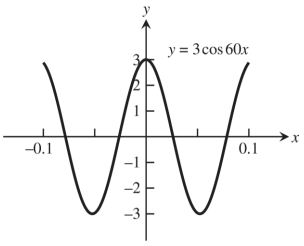
24. [−3, 5] by [−2, 10]



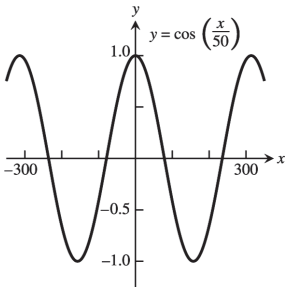
25. [−0.03, 0.03] by [−1.25, 1.25]



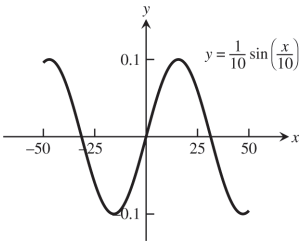
26. [−0.1, 0.1] by [−3, 3]



27. [−300, 300] by [−1.25, 1.25]

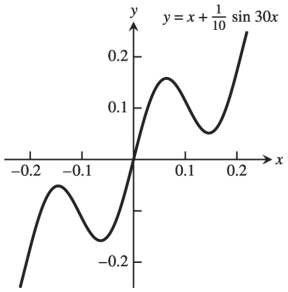


28. [−50, 50] by [−0.1, 0.1]

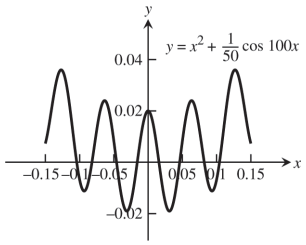


Copyright © 2020 Pearson Education, Inc.

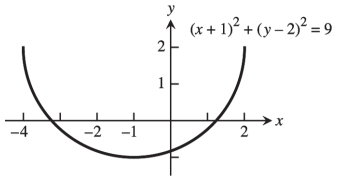
29. [−0.25, 0.25] by [−0.3, 0.3]



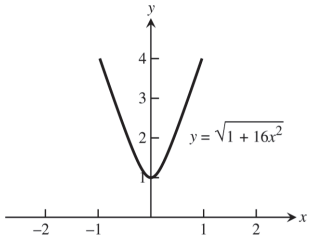
30. [−0.15, 0.15] by [−0.02, 0.05]



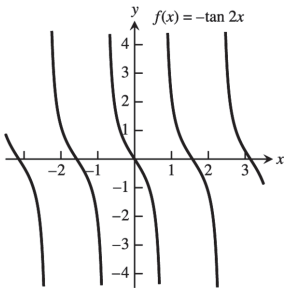
31. $x^2 + 2x = 4 + 4y - y^2 \Rightarrow y = 2 \pm \sqrt{-x^2 - 2x + 8}$. The lower half is produced by graphing $y = 2 - \sqrt{-x^2 - 2x + 8}$.



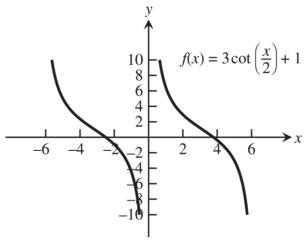
32. $y^2 - 16x^2 = 1 \Rightarrow y = \pm\sqrt{1 + 16x^2}$. The upper branch is produced by graphing $y = \sqrt{1 + 16x^2}$.



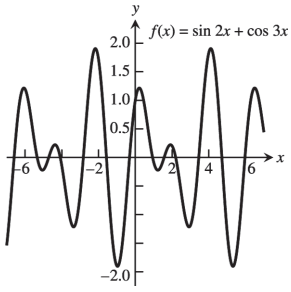
33.



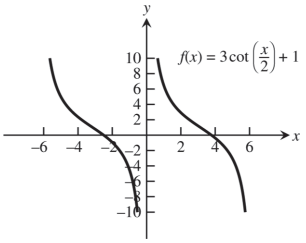
34.



35.



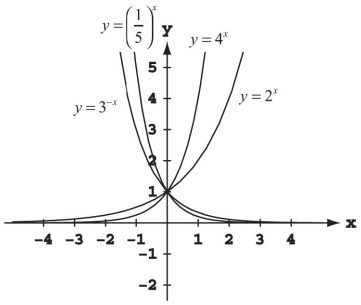
36.



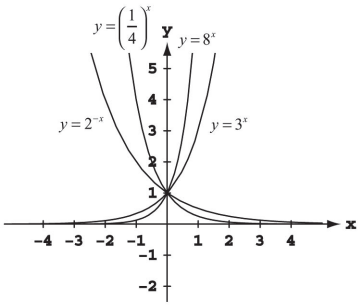
Copyright © 2020 Pearson Education, Inc.

1.5 EXPONENTIAL FUNCTIONS

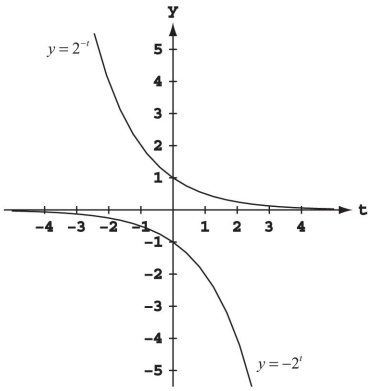
1.



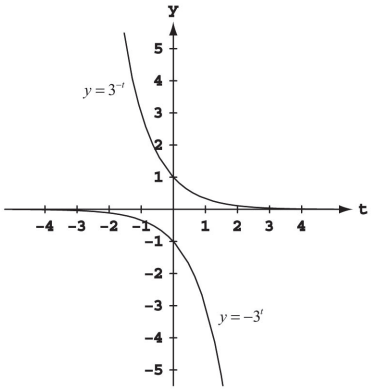
2.



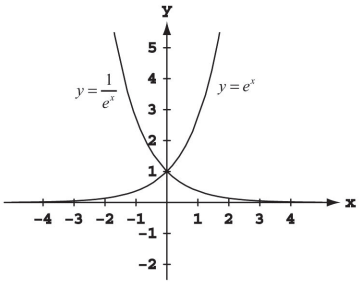
3.



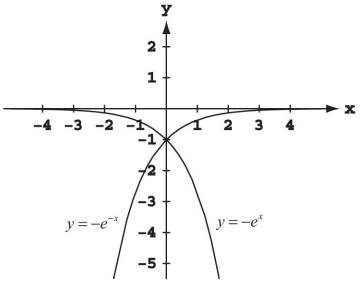
4.



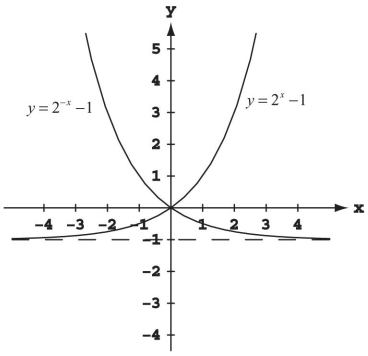
5.



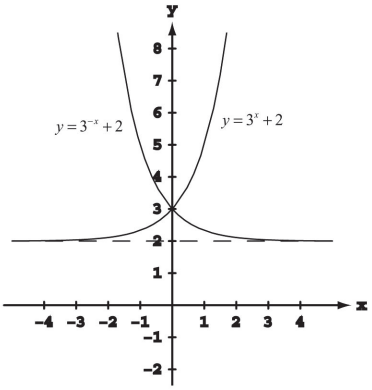
6.



7.

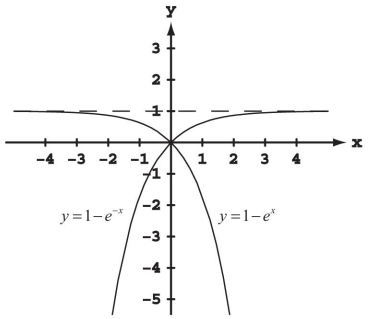


8.

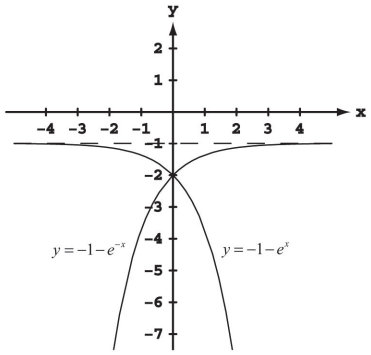


Copyright © 2020 Pearson Education, Inc.

9.



10.



11. $16^2 \cdot 16^{-1.75} = 16^{2+(-1.75)} = 16^{0.25} = 16^{1/4} = 2$

12. $9^{1/3} \cdot 9^{1/6} = 9^{\frac{1}{3}+\frac{1}{6}} = 9^{1/2} = 3$

13. $\frac{4^{4.2}}{4^{3.7}} = 4^{4.2-3.7} = 4^{0.5} = 4^{1/2} = 2$

14. $\frac{3^{5/3}}{3^{2/3}} = 3^{\frac{5}{3}-\frac{2}{3}} = 3^1 = 3$

15. $(25^{1/8})^4 = 25^{4/8} = 25^{1/2} = 5$

16. $\left(13^{\sqrt{2}}\right)^{\sqrt{2}/2} = 13^{2/2} = 13$

17. $2^{\sqrt{3}} \cdot 7^{\sqrt{3}} = (2 \cdot 7)^{\sqrt{3}} = 14^{\sqrt{3}}$

18. $\left(\sqrt{3}\right)^{1/2}\left(\sqrt{12}\right)^{1/2} = \left(\sqrt{3} \cdot \sqrt{12}\right)^{1/2} = \left(\sqrt{36}\right)^{1/2} = 6^{1/2}$

19. $\left(\frac{2}{\sqrt{2}}\right)^4 = \frac{2^4}{(2^{1/2})^4} = \frac{16}{2^2} = 4$

20. $\left(\frac{\sqrt{6}}{3}\right)^2 = \frac{(6^{1/2})^2}{3^2} = \frac{6}{9} = \frac{2}{3}$

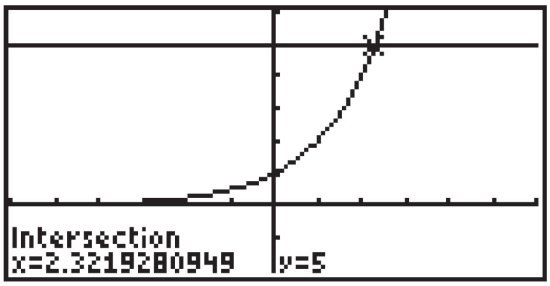
21. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \frac{1}{2+e^x}$. As x increases, e^x becomes infinitely large and y becomes a smaller and smaller positive real number. As x decreases, e^x becomes a smaller and smaller positive real number, $y < \frac{1}{2}$, and y gets arbitrarily close to $\frac{1}{2} \Rightarrow$ Range: $\left(0, \frac{1}{2}\right)$.

22. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \cos(e^{-t})$. Since the values of e^{-t} are $(0, \infty)$ and $-1 \le \cos x \le 1 \Rightarrow$ Range: $[-1, 1]$.

23. Domain: $(-\infty, \infty)$; y in range $\Rightarrow y = \sqrt{1+3^{-t}}$. Since the values of 3^{-t} are $(0, \infty) \Rightarrow$ Range: $(1, \infty)$.

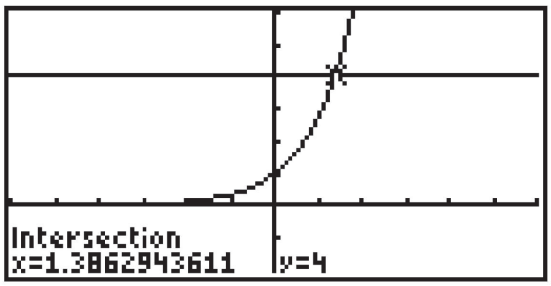
24. If $e^{2x} = 1$, then $x = 0 \Rightarrow$ Domain: $(-\infty, 0) \cup (0, \infty)$; y in range $\Rightarrow y = \frac{3}{1-e^{2x}}$. If $x > 0$, then $1 < e^{2x} < \infty$ $\Rightarrow -\infty < y < 0$. If $x < 0$, then $0 < e^{2x} < 1 \Rightarrow 3 < y < \infty \Rightarrow$ Range: $(-\infty, 0) \cup (3, \infty)$.

Copyright © 2020 Pearson Education, Inc.

25. 

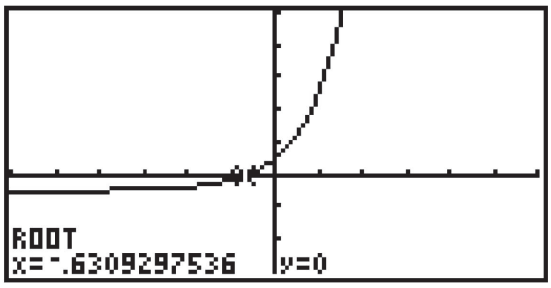
[-6, 6] by [-2, 6]

$x \approx 2.3219$

26. 

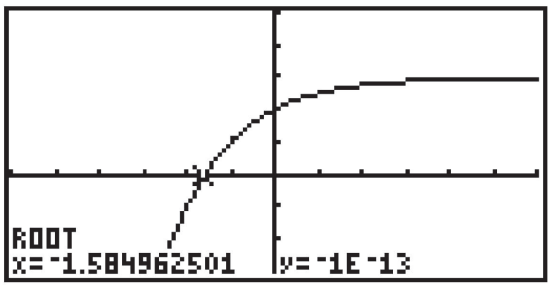
[-6, 6] by [-2, 6]

$x \approx 1.3863$

27. 

[-6, 6] by [-3, 5]

$x \approx -0.6309$

28. 

[-6, 6] by [-3, 5]

$x \approx -1.5850$

29. Let t be the number of years. Solving $500{,}000(1.0375)^t = 1{,}000{,}000$ graphically, we find that $t \approx 18.828$. The population will reach 1 million in about 19 years.

30. (a) The population is given by $P(t) = 6250(1.0275)^t$, where t is the number of years after 1890.
Population in 1915: $P(25) \approx 12{,}315$
Population in 1940: $P(50) \approx 24{,}265$
(b) Solving $P(t) = 50{,}000$ graphically, we find that $t \approx 76.651$. The population reached 50,000 about 77 years after 1890, in 1967.

31. (a) $A(t) = 6.6\left(\frac{1}{2}\right)^{t/14}$
(b) Solving $A(t) = 1$ graphically, we find that $t \approx 38$. There will be 1 gram remaining after about 38.1145 days.

32. Let t be the number of years. Solving $2300(1.60)^t = 4150$ graphically, we find that $t \approx 10.129$. It will take about 10.129 years. (If the interest is not credited to the account until the end of each year, it will take 11 years.)

33. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $A(1.0625)^t = 2A$, which is equivalent to $1.0625^t = 2$. Solving graphically, we find that $t \approx 11.433$. It will take about 11.433 years. (If the interest is credited at the end of each year, it will take 12 years.)

34. Let A be the amount of the initial investment, and let t be the number of years. We wish to solve $Ae^{0.0575t} = 3A$, which is equivalent to $e^{0.0575t} = 3$. Solving graphically, we find that $t \approx 19.106$. It will take about 19.106 years.

Copyright © 2020 Pearson Education, Inc.

35. After t hours, the population is $P(t) = 2^{t/0.5}$, or equivalently, $P(t) = 2^{2t}$. After 24 hours, the population is $P(24) = 2^{48} \approx 2.815 \times 10^{14}$ bacteria.

36. (a) Each year, the number of cases is 100% − 20% = 80% of the previous year's number of cases. After t years, the number of cases will be $C(t) = 10{,}000(0.8)^t$. Solving $C(t) = 1000$ graphically, we find that $t \approx 10.319$. It will take 10.319 years.
(b) Solving $C(t) = 1$ graphically, we find that $t \approx 41.275$. It will take about 41.275 years.

1.6 INVERSE FUNCTIONS AND LOGARITHMS

1. Yes one-to-one, the graph passes the horizontal line test.

2. Not one-to-one, the graph fails the horizontal line test.

3. Not one-to-one since (for example) the horizontal line $y = 2$ intersects the graph twice.

4. Not one-to-one, the graph fails the horizontal line test.

5. Yes one-to-one, the graph passes the horizontal line test.

6. Yes one-to-one, the graph passes the horizontal line test.

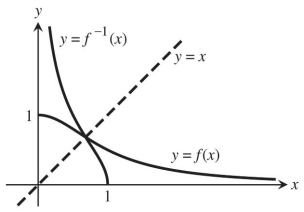
7. Not one-to-one since the horizontal line $y = 3$ intersects the graph an infinite number of times.

8. Yes one-to-one, the graph passes the horizontal line test.

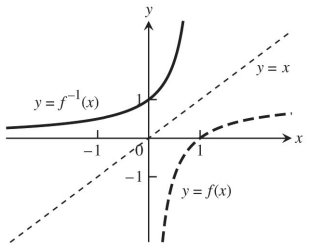
9. Yes one-to one, graph passes the horizontal line test.

10. Not one-to-one since (for example) the horizontal line $y = 1$ intersects the graph twice.

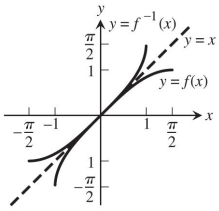
11. Domain: $0 < x \le 1$, Range: $0 \le y$



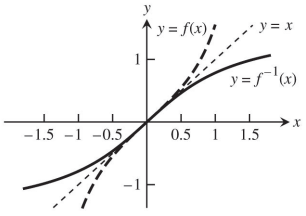
12. Domain: $x < 1$, Range: $y > 0$



13. Domain: $-1 \le x \le 1$, Range: $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

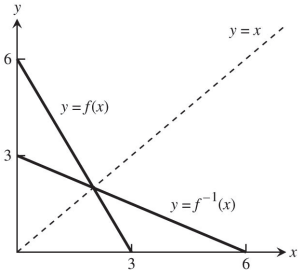


14. Domain: $-\infty < x < \infty$, Range: $-\frac{\pi}{2} < y \le \frac{\pi}{2}$

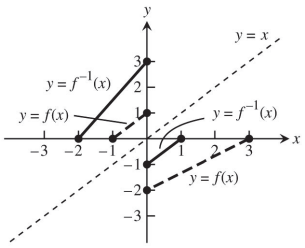


Copyright © 2020 Pearson Education, Inc.

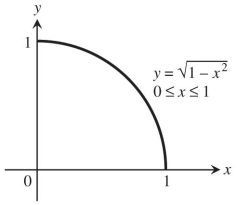
15. Domain: $0 \le x \le 6$, Range: $0 \le y \le 3$



16. Domain: $-2 \le x \le 1$, Range: $-1 \le y < 3$



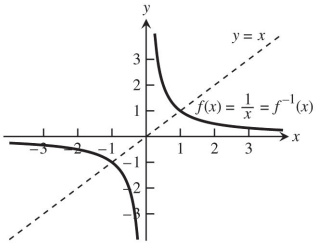
17. (a) The graph is symmetric about $y = x$.



(b) $y = \sqrt{1-x^2} \Rightarrow y^2 = 1 - x^2 \Rightarrow x^2 = 1 - y^2 \Rightarrow x = \sqrt{1-y^2} \Rightarrow y = \sqrt{1-x^2} = f^{-1}(x)$

18. (a) The graph is symmetric about $y = x$.

(b) $y = \frac{1}{x} \Rightarrow x = \frac{1}{y} \Rightarrow y = \frac{1}{x} = f^{-1}(x)$



19. Step 1: $y = x^2 + 1 \Rightarrow x^2 = y - 1 \Rightarrow x = \sqrt{y-1}$
Step 2: $y = \sqrt{x-1} = f^{-1}(x)$

20. Step 1: $y = x^2 \Rightarrow x = -\sqrt{y}$, since $x \le 0$.
Step 2: $y = -\sqrt{x} = f^{-1}(x)$

21. Step 1: $y = x^3 - 1 \Rightarrow x^3 = y + 1 \Rightarrow x = (y+1)^{1/3}$
Step 2: $y = \sqrt[3]{x+1} = f^{-1}(x)$

22. Step 1: $y = x^2 - 2x + 1 \Rightarrow y = (x-1)^2 \Rightarrow \sqrt{y} = x - 1$, since $x \ge 1 \Rightarrow x = 1 + \sqrt{y}$
Step 2: $y = 1 + \sqrt{x} = f^{-1}(x)$

23. Step 1: $y = (x+1)^2 \Rightarrow \sqrt{y} = x + 1$, since $x \ge -1 \Rightarrow x = \sqrt{y} - 1$
Step 2: $y = \sqrt{x} - 1 = f^{-1}(x)$

Copyright © 2020 Pearson Education, Inc.

24. Step 1: $y = x^{2/3} \Rightarrow x = y^{3/2}$

Step 2: $y = x^{3/2} = f^{-1}(x)$

25. Step 1: $y = x^5 \Rightarrow x = y^{1/5}$

Step 2: $y = \sqrt[5]{x} = f^{-1}(x)$;

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = (x^{1/5})^5 = x$ and $f^{-1}(f(x)) = (x^5)^{1/5} = x$

26. Step 1: $y = x^4 \Rightarrow x = y^{1/4}$

Step 2: $y = \sqrt[4]{x} = f^{-1}(x)$;

Domain of f^{-1}: $x \geq 0$, Range of f^{-1}: $y \geq 0$;

$f(f^{-1}(x)) = (x^{1/4})^4 = x$ and $f^{-1}(f(x)) = (x^4)^{1/4} = x$

27. Step 1: $y = x^3 + 1 \Rightarrow x^3 = y - 1 \Rightarrow x = (y-1)^{1/3}$

Step 2: $y = \sqrt[3]{x-1} = f^{-1}(x)$;

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = ((x-1)^{1/3})^3 + 1 = (x-1) + 1 = x$ and $f^{-1}(f(x)) = ((x^3+1)-1)^{1/3} = (x^3)^{1/3} = x$

28. Step 1: $y = \frac{1}{2}x - \frac{7}{2} \Rightarrow \frac{1}{2}x = y + \frac{7}{2} \Rightarrow x = 2y + 7$

Step 2: $y = 2x + 7 = f^{-1}(x)$;

Domain and Range of f^{-1}: all reals;

$f(f^{-1}(x)) = \frac{1}{2}(2x+7) - \frac{7}{2} = \left(x + \frac{7}{2}\right) - \frac{7}{2} = x$ and $f^{-1}(f(x)) = 2\left(\frac{1}{2}x - \frac{7}{2}\right) + 7 = (x-7) + 7 = x$

29. Step 1: $y = \frac{1}{x^2} \Rightarrow x^2 = \frac{1}{y} \Rightarrow x = \frac{1}{\sqrt{y}}$

Step 2: $y = \frac{1}{\sqrt{x}} = f^{-1}(x)$

Domain of f^{-1}: $x > 0$, Range of f^{-1}: $y > 0$;

$f(f^{-1}(x)) = \dfrac{1}{\left(\frac{1}{\sqrt{x}}\right)^2} = \dfrac{1}{\left(\frac{1}{x}\right)} = x$ and $f^{-1}(f(x)) = \dfrac{1}{\sqrt{\frac{1}{x^2}}} = \dfrac{1}{\left(\frac{1}{x}\right)} = x$ since $x > 0$.

30. Step 1: $y = \frac{1}{x^3} \Rightarrow x^3 = \dfrac{1}{y} \Rightarrow x = \dfrac{1}{y^{1/3}}$

Step 2: $y = \dfrac{1}{x^{1/3}} = \sqrt[3]{\frac{1}{x}} = f^{-1}(x)$;

Domain of f^{-1}: $x \neq 0$, Range of f^{-1}: $y \neq 0$;

$f(f^{-1}(x)) = \dfrac{1}{(x^{-1/3})^3} = \dfrac{1}{x^{-1}} = x$ and $f^{-1}(f(x)) = \left(\frac{1}{x^3}\right)^{-1/3} = \left(\frac{1}{x}\right)^{-1} = x$

Copyright © 2020 Pearson Education, Inc.

31. Step 1: $y = \frac{x+3}{x-2} \Rightarrow y(x-2) = x+3 \Rightarrow xy - 2y = x+3 \Rightarrow xy - x = 2y+3 \Rightarrow x = \frac{2y+3}{y-1}$

Step 2: $y = \frac{2x+3}{x-1} = f^{-1}(x)$;

Domain of f^{-1}: $x \neq 1$, Range of f^{-1}: $y \neq 2$;

$f(f^{-1}(x)) = \frac{\left(\frac{2x+3}{x-1}\right)+3}{\left(\frac{2x+3}{x-1}\right)-2} = \frac{(2x+3)+3(x-1)}{(2x+3)-2(x-1)} = \frac{5x}{5} = x$ and $f^{-1}(f(x)) = \frac{2\left(\frac{x+3}{x-2}\right)+3}{\left(\frac{x+3}{x-2}\right)-1} = \frac{2(x+3)+3(x-2)}{(x+3)-(x-2)} = \frac{5x}{5} = x$

32. Step 1: $y = \frac{\sqrt{x}}{\sqrt{x}-3} \Rightarrow y\left(\sqrt{x}-3\right) = \sqrt{x} \Rightarrow y\sqrt{x} - 3y = \sqrt{x} \Rightarrow y\sqrt{x} - \sqrt{x} = 3y \Rightarrow x = \left(\frac{3y}{y-1}\right)^2$

Step 2: $y = \left(\frac{3x}{x-1}\right)^2 = f^{-1}(x)$;

Domain of f^{-1}: $(-\infty, 0] \cup (1, \infty)$, Range of f^{-1}: $[0, 9) \cup (9, \infty)$;

$f(f^{-1}(x)) = \frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2 - 3}}$; If $x > 1$ or $x \leq 0 \Rightarrow \frac{3x}{x-1} \geq 0 \Rightarrow \frac{\sqrt{\left(\frac{3x}{x-1}\right)^2}}{\sqrt{\left(\frac{3x}{x-1}\right)^2 - 3}} = \frac{\frac{3x}{x-1}}{\frac{3x}{x-1} - 3} = \frac{3x}{3x-3(x-1)} = \frac{3x}{3} = x$ and

$f^{-1}(f(x)) = \left(\frac{3\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right)}{\frac{\sqrt{x}}{\left(\frac{\sqrt{x}}{\sqrt{x}-3}\right)-1}}\right)^2 = \frac{9x}{\left(\sqrt{x}-\left(\sqrt{x}-3\right)\right)^2} = \frac{9x}{9} = x$

33. Step 1: $y = x^2 - 2x,\ x \leq 1 \Rightarrow y+1 = (x-1)^2,\ x \leq 1 \Rightarrow -\sqrt{y+1} = x-1,\ x \leq 1 \Rightarrow x = 1 - \sqrt{y+1}$

Step 2: $y = 1 - \sqrt{x+1} = f^{-1}(x)$;

Domain of f^{-1}: $[-1, \infty)$, Range of f^{-1}: $(-\infty, 1]$;

$f(f^{-1}(x)) = \left(1-\sqrt{x+1}\right)^2 - 2\left(1-\sqrt{x+1}\right) = 1 - 2\sqrt{x+1} + x + 1 - 2 + 2\sqrt{x+1} = x$ and

$f^{-1}(f(x)) = 1 - \sqrt{(x^2-2x)+1},\ \ x \leq 1 = 1 - \sqrt{(x-1)^2},\ x \leq 1 = 1 - |x-1| = 1 - (1-x) = x$

34. Step 1: $y = (2x^3+1)^{1/5} \Rightarrow y^5 = 2x^3 + 1 \Rightarrow y^5 - 1 = 2x^3 \Rightarrow \frac{y^5-1}{2} = x^3 \Rightarrow x = \sqrt[3]{\frac{y^5-1}{2}}$

Step 2: $y = \sqrt[3]{\frac{x^5-1}{2}} = f^{-1}(x)$;

Domain of f^{-1}: $(-\infty, \infty)$, Range of f^{-1}: $(-\infty, \infty)$;

$f(f^{-1}(x)) = \left(2\left(\sqrt[3]{\frac{x^5-1}{2}}\right)^3 + 1\right)^{1/5} = \left(2\left(\frac{x^5-1}{2}\right)+1\right)^{1/5} = ((x^5-1)+1)^{1/5} = (x^5)^{1/5} = x$ and

$f^{-1}(f(x)) = \sqrt[3]{\frac{\left[(2x^3+1)^{1/5}\right]^5 - 1}{2}} = \sqrt[3]{\frac{(2x^3+1)-1}{2}} = \sqrt[3]{\frac{2x^3}{2}} = x$

35. $y = \frac{x+b}{x-2} = \frac{x-2+2+b}{x-2} = 1 + \frac{2+b}{x-2}, \quad x = \frac{2y+b}{y-1},$

$f^{-1}(x) = \frac{2x+b}{x-1}$

Copyright © 2020 Pearson Education, Inc.

36. Since $x \le b$, $x^2 - 2bx - y = 0$, $x = \dfrac{2b \pm \sqrt{4b^2 + 4y}}{2} = b - \sqrt{b^2 + y}$,
$x = b - \sqrt{b^2 + y}$, $f^{-1}(x) = b + \sqrt{b^2 - x}$

37. (a) $y = mx \Rightarrow x = \frac{1}{m}y \Rightarrow f^{-1}(x) = \frac{1}{m}x$

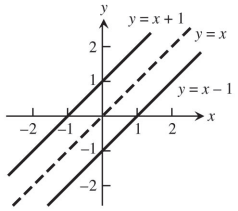
(b) The graph of $y = f^{-1}(x)$ is a line through the origin with slope $\frac{1}{m}$.

38. $y = mx + b \Rightarrow x = \frac{y}{m} - \frac{b}{m} \Rightarrow f^{-1}(x) = \frac{1}{m}x - \frac{b}{m}$; the graph of $f^{-1}(x)$ is a line with slope $\frac{1}{m}$ and y-intercept $-\frac{b}{m}$.

39. (a) $y = x + 1 \Rightarrow x = y - 1 \Rightarrow f^{-1}(x) = x - 1$

(b) $y = x + b \Rightarrow x = y - b \Rightarrow f^{-1}(x) = x - b$

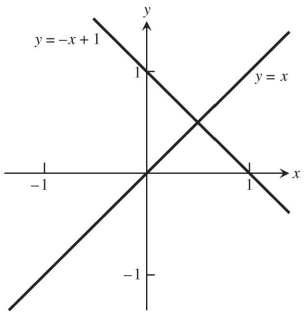
(c) Their graphs will be parallel to one another and lie on opposite sides of the line $y = x$ equidistant from that line.



40. (a) $y = -x + 1 \Rightarrow x = -y + 1 \Rightarrow f^{-1}(x) = 1 - x$;
the lines intersect at a right angle

(b) $y = -x + b \Rightarrow x = -y + b \Rightarrow f^{-1}(x) = b - x$;
the lines intersect at a right angle

(c) Such a function is its own inverse



41. (a) $\ln 0.75 = \ln \frac{3}{4} = \ln 3 - \ln 4 = \ln 3 - \ln 2^2 = \ln 3 - 2\ln 2$

(b) $\ln \frac{4}{9} = \ln 4 - \ln 9 = \ln 2^2 - \ln 3^2 = 2\ln 2 - 2\ln 3$

(c) $\ln \frac{1}{2} = \ln 1 - \ln 2 = -\ln 2$

(d) $\ln \sqrt[3]{9} = \frac{1}{3}\ln 9 = \frac{1}{3}\ln 3^2 = \frac{2}{3}\ln 3$

(e) $\ln 3\sqrt{2} = \ln 3 + \ln 2^{1/2} = \ln 3 + \frac{1}{2}\ln 2$

(f) $\ln \sqrt{13.5} = \frac{1}{2}\ln 13.5 = \frac{1}{2}\ln \frac{27}{2} = \frac{1}{2}(\ln 3^3 - \ln 2) = \frac{1}{2}(3\ln 3 - \ln 2)$

42. (a) $\ln \frac{1}{125} = \ln 1 - 3\ln 5 = -3\ln 5$

(b) $\ln 9.8 = \ln \frac{49}{5} = \ln 7^2 - \ln 5 = 2\ln 7 - \ln 5$

(c) $\ln 7\sqrt{7} = \ln 7^{3/2} = \frac{3}{2}\ln 7$

(d) $\ln 1225 = \ln 35^2 = 2\ln 35 = 2\ln 5 + 2\ln 7$

(e) $\ln 0.056 - \ln \frac{7}{125} = \ln 7 - \ln 5^3 = \ln 7 - 3\ln 5$

(f) $\frac{\ln 35 + \ln \frac{1}{7}}{\ln 25} = \frac{\ln 5 + \ln 7 - \ln 7}{2\ln 5} = \frac{1}{2}$

Copyright © 2020 Pearson Education, Inc.

43. (a) $\ln \sin\theta - \ln\left(\frac{\sin\theta}{5}\right) = \ln\left(\frac{\sin\theta}{\left(\frac{\sin\theta}{5}\right)}\right) = \ln 5$ (b) $\ln(3x^2 - 9x) + \ln\left(\frac{1}{3x}\right) = \ln\left(\frac{3x^2 - 9x}{3x}\right) = \ln(x-3)$

(c) $\frac{1}{2}\ln(4t^4) - \ln b = \ln\sqrt{4t^4} - \ln b = \ln 2t^2 - \ln b = \ln\left(\frac{2t^2}{b}\right)$

44. (a) $\ln \sec\theta + \ln\cos\theta = \ln[(\sec\theta)(\cos\theta)] = \ln 1 = 0$

(b) $\ln(8x+4) - \ln 2^2 = \ln(8x+4) - \ln 4 = \ln\left(\frac{8x+4}{4}\right) = \ln(2x+1)$

(c) $3\ln\sqrt[3]{t^2-1} - \ln(t+1) = 3\ln(t^2-1)^{1/3} - \ln(t+1) = 3\left(\frac{1}{3}\right)\ln(t^2-1) - \ln(t+1) = \ln\left(\frac{(t+1)(t-1)}{(t+1)}\right) = \ln(t-1)$

45. (a) $e^{\ln 7.2} = 7.2$ (b) $e^{-\ln x^2} = \frac{1}{e^{\ln x^2}} = \frac{1}{x^2}$ (c) $e^{\ln x - \ln y} = e^{\ln(x/y)} = \frac{x}{y}$

46. (a) $e^{\ln(x^2+y^2)} = x^2 + y^2$ (b) $e^{-\ln 0.3} = \frac{1}{e^{\ln 0.3}} = \frac{1}{0.3}$ (c) $e^{\ln \pi x - \ln 2} = e^{\ln(\pi x/2)} = \frac{\pi x}{2}$

47. (a) $2\ln\sqrt{e} = 2\ln e^{1/2} = (2)\left(\frac{1}{2}\right)\ln e = 1$ (b) $\ln(\ln e^e) = \ln(e \ln e) = \ln e = 1$

(c) $\ln e^{(-x^2-y^2)} = (-x^2 - y^2)\ln e = -x^2 - y^2$

48. (a) $\ln\left(e^{\sec\theta}\right) = (\sec\theta)(\ln e) = \sec\theta$ (b) $\ln e^{(e^x)} = (e^x)(\ln e) = e^x$

(c) $\ln(e^{2\ln x}) = \ln\left(e^{\ln x^2}\right) = \ln x^2 = 2\ln x$

49. $\ln y = 2t + 4 \Rightarrow e^{\ln y} = e^{2t+4} \Rightarrow y = e^{2t+4}$ 50. $\ln y = -t + 5 \Rightarrow e^{\ln y} = e^{-t+5} \Rightarrow y = e^{-t+5}$

51. $\ln(y - 40) = 5t \Rightarrow e^{\ln(y-40)} = e^{5t} \Rightarrow y - 40 = e^{5t} \Rightarrow y = e^{5t} + 40$

52. $\ln(1 - 2y) = t \Rightarrow e^{\ln(1-2y)} = e^t \Rightarrow 1 - 2y = e^t \Rightarrow -2y = e^t - 1 \Rightarrow y = -\left(\frac{e^t - 1}{2}\right)$

53. $\ln(y-1) - \ln 2 = x + \ln x \Rightarrow \ln(y-1) - \ln 2 - \ln x = x \Rightarrow \ln\left(\frac{y-1}{2x}\right) = x \Rightarrow e^{\ln\left(\frac{y-1}{2x}\right)} = e^x \Rightarrow \frac{y-1}{2x} = e^x$

$\Rightarrow y - 1 = 2xe^x \Rightarrow y = 2xe^x + 1$

54. $\ln(y^2-1) - \ln(y+1) = \ln(\sin x) \Rightarrow \ln\left(\frac{y^2-1}{y+1}\right) = \ln(\sin x) \Rightarrow \ln(y-1) = \ln(\sin x) \Rightarrow e^{\ln(y-1)} = e^{\ln(\sin x)}$

$\Rightarrow y - 1 = \sin x \Rightarrow y = \sin x + 1$

55. (a) $e^{2k} = 4 \Rightarrow \ln e^{2k} = \ln 4 \Rightarrow 2k \ln e = \ln 2^2 \Rightarrow 2k = 2\ln 2 \Rightarrow k = \ln 2$

(b) $100e^{10k} = 200 \Rightarrow e^{10k} = 2 \Rightarrow \ln e^{10k} = \ln 2 \Rightarrow 10k \ln e = \ln 2 \Rightarrow 10k = \ln 2 \Rightarrow k = \frac{\ln 2}{10}$

(c) $e^{k/1000} = a \Rightarrow \ln e^{k/1000} = \ln a \Rightarrow \frac{k}{1000}\ln e = \ln a \Rightarrow \frac{k}{1000} = \ln a \Rightarrow k = 1000 \ln a$

Copyright © 2020 Pearson Education, Inc.

56. (a) $e^{5k} = \frac{1}{4} \Rightarrow \ln e^{5k} = \ln 4^{-1} \Rightarrow 5k \ln e = -\ln 4 \Rightarrow 5k = -\ln 4 \Rightarrow k = -\frac{\ln 4}{5}$

(b) $80e^k = 1 \Rightarrow e^k = 80^{-1} \Rightarrow \ln e^k = \ln 80^{-1} \Rightarrow k \ln e = -\ln 80 \Rightarrow k = -\ln 80$

(c) $e^{(\ln 0.8)k} = 0.8 \Rightarrow (e^{\ln 0.8})^k = 0.8 \Rightarrow (0.8)^k = 0.8 \Rightarrow k = 1$

57. (a) $e^{-0.3t} = 27 \Rightarrow \ln e^{-0.3t} = \ln 3^3 \Rightarrow (-0.3t)\ln e = 3 \ln 3 \Rightarrow -0.3t = 3 \ln 3 \Rightarrow t = -10 \ln 3$

(b) $e^{kt} = \frac{1}{2} \Rightarrow \ln e^{kt} = \ln 2^{-1} = kt \ln e = -\ln 2 \Rightarrow t = -\frac{\ln 2}{k}$

(c) $e^{(\ln 0.2)t} = 0.4 \Rightarrow (e^{\ln 0.2})^t = 0.4 \Rightarrow 0.2^t = 0.4 \Rightarrow \ln 0.2^t = \ln 0.4 \Rightarrow t \ln 0.2 = \ln 0.4 \Rightarrow t = \frac{\ln 0.4}{\ln 0.2}$

58. (a) $e^{-0.01t} = 1000 \Rightarrow \ln e^{-0.01t} = \ln 1000 \Rightarrow (-0.01\text{t})\ln e = \ln 1000 \Rightarrow -0.01t = \ln 1000 \Rightarrow t = -100 \ln 1000$

(b) $e^{kt} = \frac{1}{10} \Rightarrow \ln e^{kt} = \ln 10^{-1} = kt \ln e = -\ln 10 \Rightarrow kt = -\ln 10 \Rightarrow t = -\frac{\ln 10}{k}$

(c) $e^{(\ln 2)t} = \frac{1}{2} \Rightarrow (e^{\ln 2})^t = 2^{-1} \Rightarrow 2^t = 2^{-1} \Rightarrow t = -1$

59. $e^{\sqrt{t}} = x^2 \Rightarrow \ln e^{\sqrt{t}} = \ln x^2 \Rightarrow \sqrt{t} = 2\ln|x| \Rightarrow t = 4\left(\ln|x|\right)^2$

60. $e^{x^2} e^{2x+1} = e^t \Rightarrow e^{x^2+2x+1} = e^t \Rightarrow \ln e^{x^2+2x+1} = \ln e^t \Rightarrow t = x^2 + 2x + 1$

61. $e^{2t} - 3e^t = (e^t)^2 - 3e^t = e^t(e^t - 3) = 0 \Rightarrow e^t - 3 = 0 \Rightarrow e^t = 3 \Rightarrow \ln e^t = \ln 3 \Rightarrow t = \ln 3$

62. $e^{-2t} + 6 = 5e^{-t} \Rightarrow (e^{-t})^2 - 5e^{-t} + 6 = 0 \Rightarrow (e^{-t} - 3)(e^{-t} - 2) = 0 \Rightarrow e^{-t} - 3 = 0$ or $e^{-t} - 2 = 0 \Rightarrow e^{-t} = 3$ or $e^{-t} = 2 \Rightarrow \ln e^{-t} = \ln 3$ or $\ln e^{-t} = \ln 2 \Rightarrow -t = \ln 3$ or $-t = \ln 2 \Rightarrow t = -\ln 3$ or $t = -\ln 2$

63. $\ln\left(\frac{t}{t-1}\right) = 2 \Rightarrow e^{\ln\left(\frac{t}{t-1}\right)} = e^2 \Rightarrow \frac{t}{t-1} = e^2 \Rightarrow t = e^2 t - e^2 \Rightarrow e^2 = e^2 t - t = (e^2 - 1)t \Rightarrow t = \frac{e^2}{e^2 - 1}$

64. $\ln(t-2) = \ln 8 - \ln t \Rightarrow \ln(t-2) + \ln t = \ln 8 \Rightarrow \ln((t-2)t) = \ln 8 \Rightarrow e^{\ln(t^2-2t)} = e^{\ln 8} \Rightarrow t^2 - 2t = 8 \Rightarrow$
$t^2 - 2t - 8 = (t-4)(t+2) = 0 \Rightarrow t - 4 = 0 \Rightarrow t = 4$

65. (a) $5^{\log_5 7} = 7$ (b) $8^{\log_8 \sqrt{2}} = \sqrt{2}$ (c) $1.3^{\log_3 75} = 75$

(d) $\log_4 16 = \log_4 4^2 = 2\log_4 4 = 2 \cdot 1 = 2$

(e) $\log_3 \sqrt{3} = \log_3 3^{1/2} = \frac{1}{2}\log_3 3 = \frac{1}{2} \cdot 1 = \frac{1}{2} = 0.5$

(f) $\log_4\left(\frac{1}{4}\right) = \log_4 4^{-1} = -1\log_4 4 = -1 \cdot 1 = -1$

66. (a) $2^{\log_2 3} = 3$ (b) $10^{\log_{10}(1/2)} = \frac{1}{2}$ (c) $\pi^{\log_\pi 7} = 7$

(d) $\log_{11} 121 = \log_{11} 11^2 = 2\log_{11} 11 = 2 \cdot 1 = 2$

(e) $\log_{121} 11 = \log_{121} 121^{1/2} = \left(\frac{1}{2}\right)\log_{121} 121 = \left(\frac{1}{2}\right) \cdot 1 = \frac{1}{2}$

(f) $\log_3\left(\frac{1}{9}\right) = \log_3 3^{-2} = -2\log_3 3 = -2 \cdot 1 = -2$

Copyright © 2020 Pearson Education, Inc.

67. (a) Let $z = \log_4 x \Rightarrow 4^z = x \Rightarrow 2^{2z} = x \Rightarrow (2^z)^2 = x \Rightarrow 2^z = \sqrt{x}$

(b) Let $z = \log_3 x \Rightarrow 3^z = x \Rightarrow (3^z)^2 = x^2 \Rightarrow 3^{2z} = x^2 \Rightarrow 9^z = x^2$

(c) $\log_2(e^{(\ln 2)\sin x}) = \log_2 2^{\sin x} = \sin x$

68. (a) Let $z = \log_5(3x^2) \Rightarrow 5^z = 3x^2 \Rightarrow 25^z = 9x^4$

(b) $\log_e(e^x) = x$

(c) $\log_4(2^{e^x \sin x}) = \log_4 4^{(e^x \sin x)/2} = \frac{e^x \sin x}{2}$

69. (a) $\frac{\log_2 x}{\log_3 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 3}{\ln x} = \frac{\ln 3}{\ln 2}$ (b) $\frac{\log_2 x}{\log_8 x} = \frac{\ln x}{\ln 2} \div \frac{\ln x}{\ln 8} = \frac{\ln x}{\ln 2} \cdot \frac{\ln 8}{\ln x} = \frac{3\ln 2}{\ln 2} = 3$

(c) $\frac{\ln_x a}{\ln_{x^2} a} = \frac{\ln a}{\ln x} \div \frac{\ln a}{\ln x^2} = \frac{\ln a}{\ln x} \cdot \frac{\ln x^2}{\ln a} = \frac{2\ln x}{\ln x} = 2$

70. (a) $\frac{\log_9 x}{\log_3 x} = \frac{\ln x}{\ln 9} \div \frac{\ln x}{\ln 3} = \frac{\ln x}{2\ln 3} \cdot \frac{\ln 3}{\ln x} = \frac{1}{2}$

(b) $\frac{\log_{\sqrt{10}} x}{\log_{\sqrt{2}} x} = \frac{\ln x}{\ln\sqrt{10}} \div \frac{\ln x}{\ln\sqrt{2}} = \frac{\ln x}{\left(\frac{1}{2}\right)\ln 10} \cdot \frac{\left(\frac{1}{2}\right)\ln 2}{\ln x} = \frac{\ln 2}{\ln 10}$

(c) $\frac{\log_a b}{\log_b a} = \frac{\ln b}{\ln a} \div \frac{\ln a}{\ln b} = \frac{\ln b}{\ln a} \cdot \frac{\ln b}{\ln a} = \left(\frac{\ln b}{\ln a}\right)^2$

71. (a) $-\frac{\pi}{6}$ (b) $\frac{\pi}{4}$ (c) $-\frac{\pi}{3}$

72. (a) $\frac{\pi}{3}$ (b) $\frac{3\pi}{4}$ (c) $\frac{\pi}{6}$

73. (a) $\arccos(-1) = \pi$ since $\cos(\pi) = -1$ and $0 \le \pi \le \pi$.

(b) $\arccos(0) = \frac{\pi}{2}$ since $\cos\left(\frac{\pi}{2}\right) = 0$ and $0 \le \frac{\pi}{2} \le \pi$.

74. (a) $\arcsin(-1) = -\frac{\pi}{2}$ since $\sin\left(-\frac{\pi}{2}\right) = -1$ and $-\frac{\pi}{2} \le -\frac{\pi}{2} \le \frac{\pi}{2}$.

(b) $\arcsin\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ since $\sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ and $-\frac{\pi}{2} \le -\frac{\pi}{4} \le \frac{\pi}{2}$.

75. The function $g(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \ne x_2$ then $f(x_1) \ne f(x_2)$, so $-f(x_1) \ne -f(x_2)$ and therefore $g(x_1) \ne g(x_2)$. Therefore $g(x)$ is one-to-one as well.

76. The function $h(x)$ is also one-to-one. The reasoning: $f(x)$ is one-to-one means that if $x_1 \ne x_2$ then $f(x_1) \ne f(x_2)$, so $\frac{1}{f(x_1)} \ne \frac{1}{f(x_2)}$, and therefore $h(x_1) \ne h(x_2)$.

77. The composition is one-to-one also. The reasoning: If $x_1 \ne x_2$ then $g(x_1) \ne g(x_2)$ because g is one-to-one. Since $g(x_1) \ne g(x_2)$, we also have $f(g(x_1)) \ne f(g(x_2))$ because f is one-to-one; thus, $f \circ g$ is one-to-one because $x_1 \ne x_2 \Rightarrow f(g(x_1)) \ne f(g(x_2))$.

Copyright © 2020 Pearson Education, Inc.

78. Yes, g must be one-to-one. If g were not one-to-one, there would exist numbers $x_1 \neq x_2$ in the domain of g with $g(x_1) = g(x_2)$. For these numbers we would also have $f(g(x_1)) = f(g(x_2))$, contradicting the assumption that $f \circ g$ is one-to-one.

79. (a) $y = \frac{100}{1+2^{-x}} \to 1+2^{-x} = \frac{100}{y} \to 2^{-x} = \frac{100}{y} - 1 \to \log_2(2^{-x}) = \log_2\left(\frac{100}{y} - 1\right) \to -x = \log_2\left(\frac{100}{y} - 1\right)$

$x = -\log_2\left(\frac{100}{y} - 1\right) = -\log_2\left(\frac{100-y}{y}\right) = \log_2\left(\frac{y}{100-y}\right).$

Interchange x and y: $y = \log_2\left(\frac{x}{100-x}\right) \to f^{-1}(x) = \log_2\left(\frac{x}{100-x}\right)$

Verify.

$(f \circ f^{-1})(x) = f\left(\log_2\left(\frac{x}{100-x}\right)\right) = \frac{100}{1+2^{\log_2\left(\frac{x}{100-x}\right)}} = \frac{100}{1+2^{\log_2\left(\frac{100-x}{x}\right)}} = \frac{100}{1+\frac{100-x}{x}} = \frac{100}{x+100-x} = \frac{100x}{100} = x$

$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{100}{1+2^{-x}}\right) = \log_2\left(\frac{\frac{100}{1+2^{-x}}}{100-\frac{100}{1+2^{-x}}}\right) = \log_2\left(\frac{100}{100(1+2^{-x})-100}\right) = \log_2\left(\frac{1}{2^{-x}}\right) = \log_2(2^x) = x$

(b) $y = \frac{50}{1+1.1^{-x}} \to 1+1.1^{-x} = \frac{50}{y} \to 1.1^{-x} = \frac{50}{y} - 1 \to \log_{1.1}(1.1^{-x}) = \log_{1.1}\left(\frac{50}{y} - 1\right) \to -x = \log_{1.1}\left(\frac{50}{y} - 1\right)$

$x = -\log_{1.1}\left(\frac{50}{y} - 1\right) = -\log_{1.1}\left(\frac{50-y}{y}\right) = \log_{1.1}\left(\frac{y}{50-y}\right).$

Interchange x and y: $y = \log_{1.1}\left(\frac{x}{50-x}\right) \to f^{-1}(x) = \log_{1.1}\left(\frac{x}{50-x}\right)$

Verify.

$(f \circ f^{-1})(x) = f\left(\log_{1.1}\left(\frac{x}{50-x}\right)\right) = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{x}{50-x}\right)}} = \frac{50}{1+1.1^{\log_{1.1}\left(\frac{50-x}{x}\right)}} = \frac{50}{1+\frac{50-x}{x}} = \frac{50x}{x+50-x} = \frac{50x}{50} = x$

$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{50}{1+1.1^{-x}}\right) = \log_{1.1}\left(\frac{\frac{50}{1+1.1^{-x}}}{50-\frac{50}{1+1.1^{-x}}}\right) = \log_{1.1}\left(\frac{50}{50(1+1.1^{-x})-50}\right) = \log_{1.1}\left(\frac{1}{1.1^{-x}}\right) = \log_{1.1}(1.1^x) = x$

(c) $y = \frac{e^x-1}{e^x+1} \Rightarrow (e^x+1)y = e^x - 1 \Rightarrow e^x y + y = e^x - 1 \Rightarrow y + 1 = e^x - e^x y = (1-y)e^x \Rightarrow e^x = \frac{y+1}{1-y} \Rightarrow$

$\ln e^x = \ln\left(\frac{y+1}{1-y}\right) \Rightarrow x = \ln\left(\frac{y+1}{1-y}\right).$

Interchange x and y: $y = \ln\left(\frac{x+1}{1-x}\right) \Rightarrow f^{-1}(x) = \ln\left(\frac{x+1}{1-x}\right).$

Verify.

$(f \circ f^{-1})(x) = f\left(\ln\left(\frac{x+1}{1-x}\right)\right) = \frac{e^{\ln\left(\frac{x+1}{1-x}\right)}-1}{e^{\ln\left(\frac{x+1}{1-x}\right)}+1} = \frac{\frac{x+1}{1-x} - \frac{1-x}{1-x}}{\frac{x+1}{1-x} + \frac{1-x}{1-x}} = \frac{x+1-1+x}{x+1+1-x} = \frac{2x}{1-x} \cdot \frac{1-x}{2} = x.$

$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{e^x-1}{e^x+1}\right) = \ln\left(\frac{\frac{e^x-1}{e^x+1}+1}{1-\frac{e^x-1}{e^x+1}}\right) = \ln\left(\frac{e^x-1+(e^x+1)}{e^x+1-(e^x-1)}\right) = \ln\left(\frac{2e^x}{2}\right) = \ln e^x = x$

(d) $y = \frac{\ln x}{2-\ln x} \Rightarrow (2-\ln x)y = \ln x \Rightarrow 2y - y\ln x = \ln x \Rightarrow 2y = y\ln x + \ln x \Rightarrow 2y = (y+1)\ln x \Rightarrow \ln x = \frac{2y}{y+1} \Rightarrow$

$e^{\ln x} = e^{\frac{2y}{y+1}} \Rightarrow x = e^{\frac{2y}{y+1}}$

Interchange x and y: $y = e^{\frac{2x}{x+1}} \Rightarrow f^{-1}(x) = e^{\frac{2x}{x+1}}.$

Copyright © 2020 Pearson Education, Inc.

Verify.

$$(f \circ f^{-1})(x) = f\left(e^{\frac{2x}{x+1}}\right) = \frac{\ln\left(e^{\frac{2x}{x+1}}\right)}{2 - \ln\left(e^{\frac{2x}{x+1}}\right)} = \frac{\frac{2x}{x+1}}{2 - \frac{2x}{x+1}} \cdot \frac{x+1}{x+1} = \frac{2x}{2(x+1) - 2x} = \frac{2x}{2} = x.$$

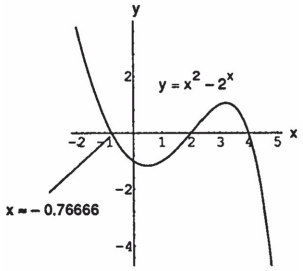
$$(f^{-1} \circ f)(x) = f^{-1}\left(\frac{\ln x}{2 - \ln x}\right) = e^{\left[\frac{2\left(\frac{\ln x}{2-\ln x}\right)}{\left(\frac{\ln x}{2-\ln x}\right)+1}\right]} = e^{\left[\frac{2\ln x}{\ln x + (2-\ln x)}\right]} = e^{\frac{2\ln x}{2}} = e^{\ln x} = x$$

80. $\sin^{-1}(1) + \cos^{-1}(1) = \frac{\pi}{2} + 0 = \frac{\pi}{2}$; $\sin^{-1}(0) + \cos^{-1}(0) = 0 + \frac{\pi}{2} = \frac{\pi}{2}$; and $\sin^{-1}(-1) + \cos^{-1}(-1) = -\frac{\pi}{2} + \pi = \frac{\pi}{2}$.
If $x \in (-1, 0)$ and $x = -a$, then $\sin^{-1}(x) + \cos^{-1}(x) = \sin^{-1}(-a) + \cos^{-1}(-a) = -\sin^{-1} a + (\pi - \cos^{-1} a)$
$= \pi - (\sin^{-1} a + \cos^{-1} a) = \pi - \frac{\pi}{2} = \frac{\pi}{2}$ from Equations (3) and (4) in the text.

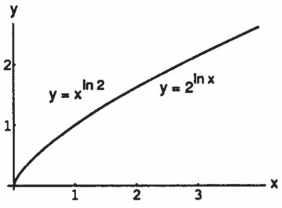
81. (a) Begin with $y = \ln x$ and reduce the y-value by 3 $\Rightarrow y = \ln x - 3$.
(b) Begin with $y = \ln x$ and replace x with $x - 1 \Rightarrow y = \ln(x - 1)$.
(c) Begin with $y = \ln x$, replace x with $x + 1$, and increase the y-value by 3 $\Rightarrow y = \ln(x + 1) + 3$.
(d) Begin with $y = \ln x$, reduce the y-value by 4, and replace x with $x - 2 \Rightarrow y = \ln(x - 2) - 4$.
(e) Begin with $y = \ln x$ and replace x with $-x \Rightarrow y = \ln(-x)$.
(f) Begin with $y = \ln x$ and switch x and $y \Rightarrow x = \ln y$ or $y = e^x$.

82. (a) Begin with $y = \ln x$ and multiply the y-value by 2 $\Rightarrow y = 2 \ln x$.
(b) Begin with $y = \ln x$ and replace x with $\frac{x}{3} \Rightarrow y = \ln\left(\frac{x}{3}\right)$.
(c) Begin with $y = \ln x$ and multiply the y-value by $\frac{1}{4} \Rightarrow y = \frac{1}{4}\ln x$.
(d) Begin with $y = \ln x$ and replace x with $2x \Rightarrow y = \ln 2x$.

83. From zooming in on the graph at the right, we estimate the third root to be $x \approx -0.76666$.



84. The functions $f(x) = x^{\ln 2}$ and $g(x) = 2^{\ln x}$ appear to have identical graphs for $x > 0$. This is no accident, because
$x^{\ln 2} = e^{\ln 2 \cdot \ln x} = (e^{\ln 2})^{\ln x} = 2^{\ln x}$.



85. (a) Amount $= 8\left(\frac{1}{2}\right)^{t/12}$
(b) $8\left(\frac{1}{2}\right)^{t/12} = 1 \to \left(\frac{1}{2}\right)^{t/12} = \frac{1}{8} \to \left(\frac{1}{2}\right)^{t/12} = \left(\frac{1}{2}\right)^3 \to \frac{t}{12} = 3 \to t = 36$
There will be 1 gram remaining after 36 hours.

86. $500(1.0475)^t = 1000 \to 1.0475^t = 2 \to \ln(1.0475^t) = \ln(2) \to t\ln(1.0475) = \ln(2) \to t = \frac{\ln(2)}{\ln(1.0475)} \approx 14.936$
It will take about 14.936 years. (If the interest is paid at the end of each year, it will take 15 years.)

Copyright © 2020 Pearson Education, Inc.

87. $375{,}000(1.0225)^t = 1{,}000{,}000 \to 1.0225^t = \frac{8}{3} \to \ln(1.0225^t) = \ln\left(\frac{8}{3}\right) \to t\ln(1.0225) = \ln\left(\frac{8}{3}\right)$

$\to t = \frac{\ln\left(\frac{8}{3}\right)}{\ln(1.0225)} \approx 44.081$

It will take about 44.081 years.

88. $y = y_0 e^{-0.18t}$ represents the decay equation; solving $(0.9)y_0 = y_0 e^{-0.18t} \Rightarrow t = \frac{\ln(0.9)}{-0.18} \approx 0.585$ days

Copyright © 2020 Pearson Education, Inc.